Comparison of the global gravitational effects obtained respectively from LITHO1.0 geophysical model and GOCE satellite data

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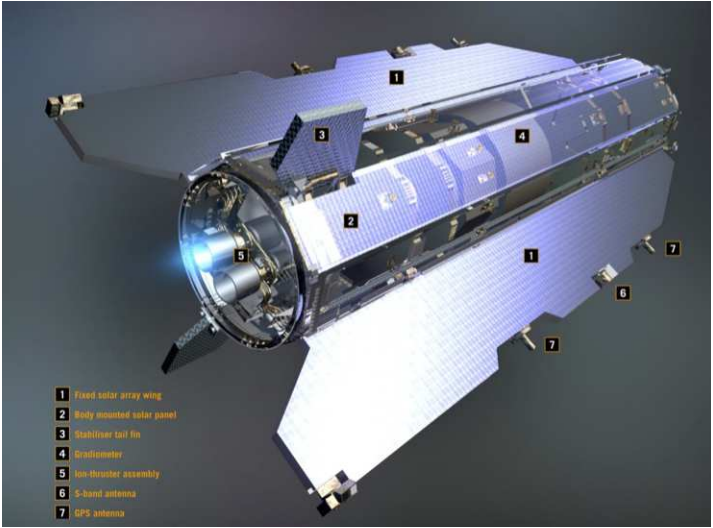
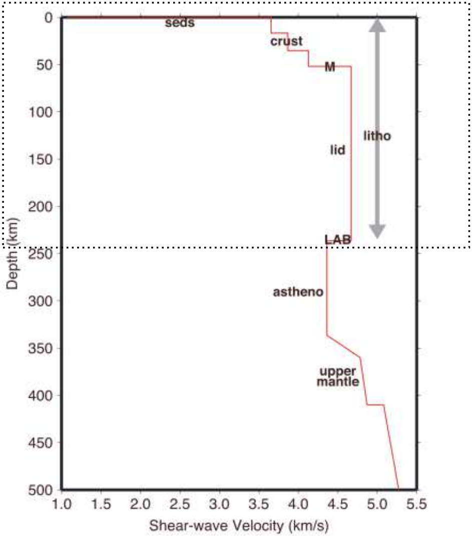


GOCE Level 2 Product Data Hanbook, EGG-C, 2014

GOCE $\Rightarrow$ full tensor gravity gradients = direct, global, high spatial resolution ($\sim$ 100 km), 1 mE accuracy



LITHO1.0

Available at 1°x1° resolution

Pasyanos, M. E., T. G. Masters, G. Laske, and Z. Ma (2014), LITHO1.0: An updated crust and lithospheric model of the Earth, J. Geophys. Res. Solid Earth, 119, 2153–2173, doi:10.1002/2013JB010626.

LITHO1.0: physical parameters

num	layer	parameters
1	ice	(thickness), (vp), (vs), (ρ), (Q)
2	water	(thickness), (vp), (vs), (ρ), (Q)
3	sediment layer 1	(thickness), (vp), (vs), (ρ), (Q)
4	sediment layer 2	(thickness), (vp), (vs), (ρ), (Q)
5	sediment layer 3	(thickness), (vp), (vs), (ρ), (Q)
6	upper crust	thickness, vp, vs, ρ, (Q)
7	middle crust	thickness, vp, vs, ρ, (Q)
8	lower crust	thickness, vp, vs, ρ, (Q)
9	lithospheric mantle (lid)	thickness, vp, vs, (ρ), (Q)
10	asthenospheric mantle	vp, vs, (ρ), (Q)

Surface

Basement

Moho

LAB

ak135

[a]Parameters unmodified from the starting model are shown in parentheses.

Pasyanos et al., 2014

Starting models

CRUST1.0

Laske et al., 2012

LLNL-G3D

Simmons et al., 2012

Pasyanos, 2010 (thickness)

Kennett et al., 1995

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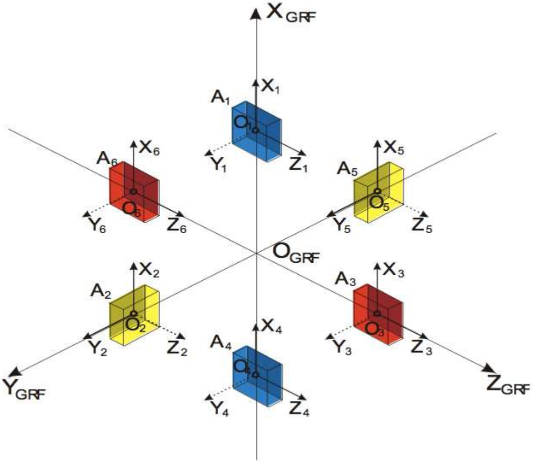
Possible comparison: FTG components deduced respectively from forward modelling and GOCE data

Outline

Brief review of GOCE reference frames

Gradiometer Reference Frame (GRF)

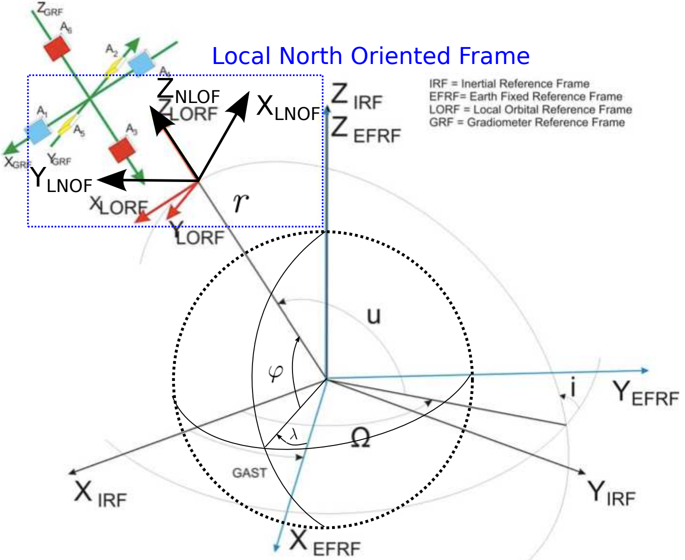


GOCE Level 2 Product Data Handbook, EGG-C, 2014

Accelerometer Reference Frames (ARF)

········➤ less sensitive axis

Brief review of GOCE reference frames



Original figure from: GOCE Level 2 Product Data Handbook, EGG-C, 2014

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Possible by means of numerical integration given a geophysical model

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1: Uieda, L., *Tesseroids: forward modeling of gravitational fields in spherical coordinates*: figshare, 2014.

2: Roussel, C., Verdun, J., Cali, J. and F. Masson, *Complete gravity field of an ellipsoidal prism in spherical coordinates by Gauss-Legendre quadrature*: in prep for submission to Geophysical J. Int., 2014.

Basic notations for the gravity field

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1. Gravitational potential: $V = \iiint_{Terre} \frac{G\,\rho(M)\,d\tau_M}{MP}$,
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   Symmetric tensor with null trace out of masses.

Spherical vs ellipsoidal prism

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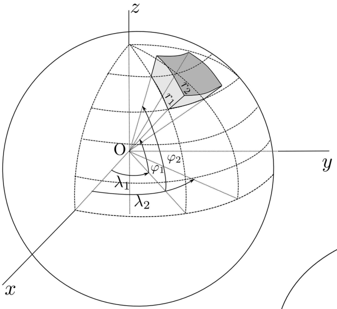
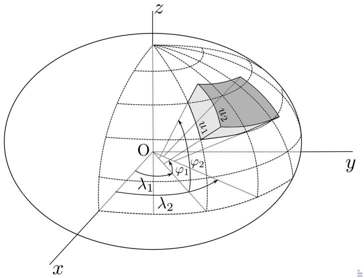


Spherical prism

Ellipsoidal prism



Gauss-Legendre Quadrature

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Prism domain

Any gravity related quantity

$$\int_{\lambda_1}^{\lambda_2}\int_{\varphi_1}^{\varphi_2}\int_{u_1}^{u_2} f(\lambda_P,\theta_P,r_P,\lambda_S,\varphi_S,u_S)\,du_S\,d\varphi_S\,d\lambda_S$$

Observation point (spherical coordinates)

Source point (ellipsoidal coordinates)

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Observation point (spherical coordinates)

Source point (ellipsoidal coordinates)

$\approx$

Prism domain

Quantity to be integrated

$$\frac{(\lambda_2-\lambda_1)(\varphi_2-\varphi_1)(u_2-u_1)}{8}\sum_{i=1}^{n_\lambda}\sum_{j=1}^{n_\varphi}\sum_{k=1}^{n_u}\omega_i\omega_j\omega_k f(\lambda_P,\theta_P,r_P,\lambda_{S_i},\varphi_{S_j},u_{S_k})$$

Observation point (spherical coordinates)

Source point at Gauss-Legendre nodes (ellipsoidal coordinates)

Gauss-Legendre weigthing coefficients

Gauss-Quadrature validation: example for V_{zz}

$e_{T_{rr}} = e_{V_{zz}} = (V_{zz})_{GLQ} - (V_{zz})_{exact\ formulas}$ for an ellipsoidal layer of constant density.

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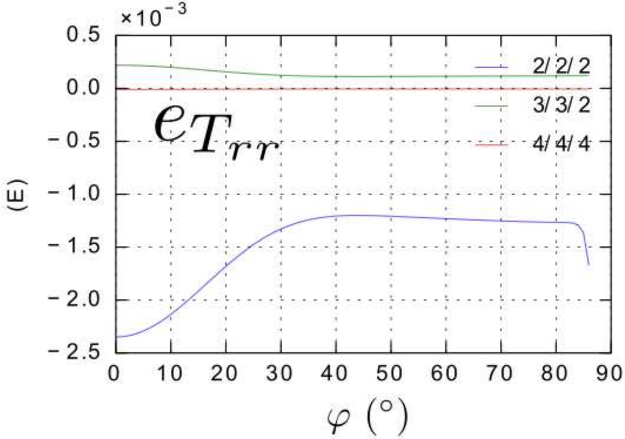
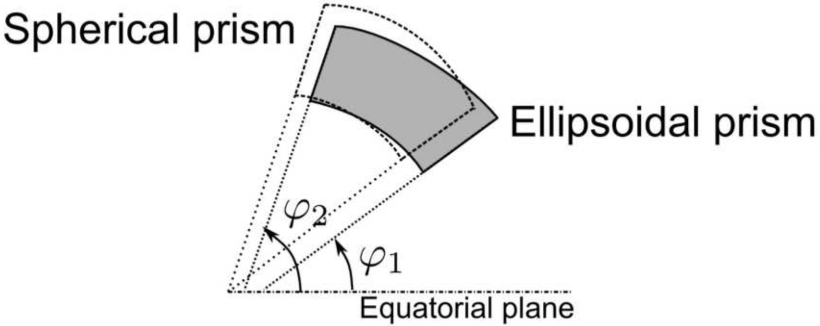


Roussel et al., 2014

To what extent the spherical approximation remains valid ?



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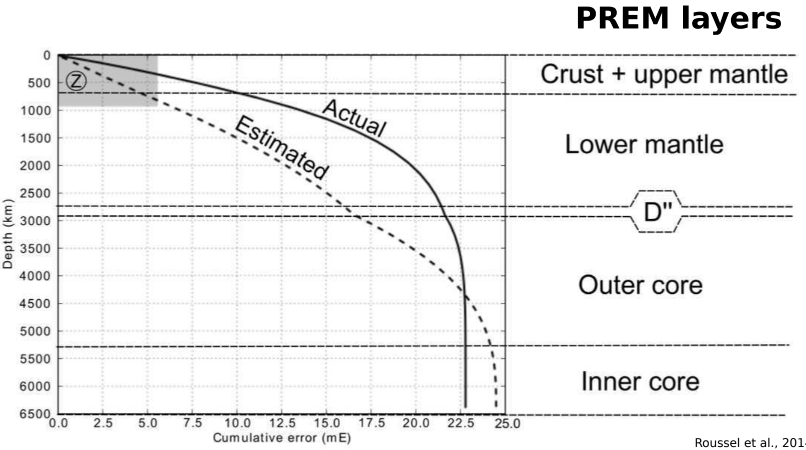

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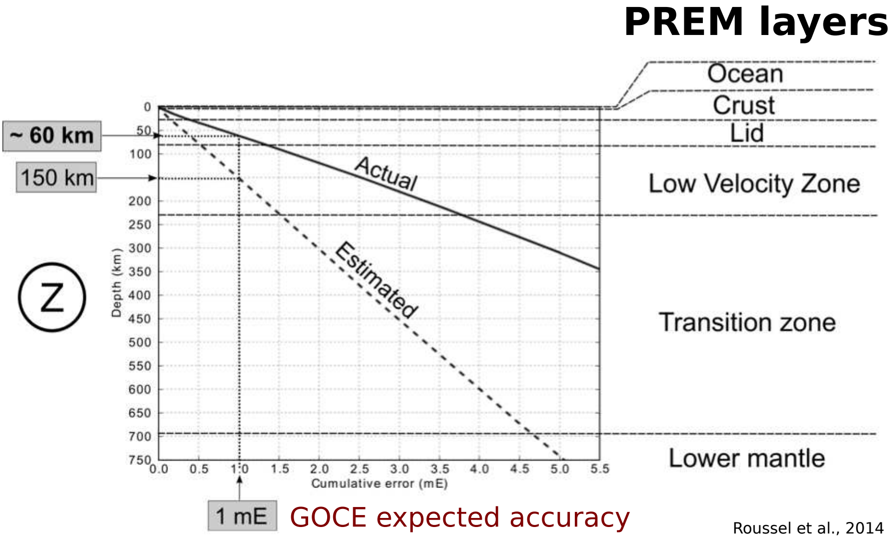


GOCE expected accuracy

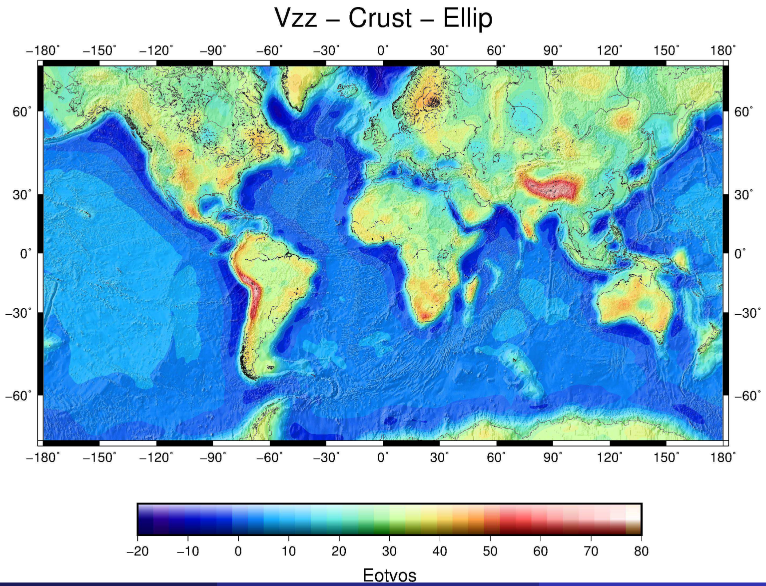
Roussel et al., 2014

To what extent the spherical approximation remains valid ?

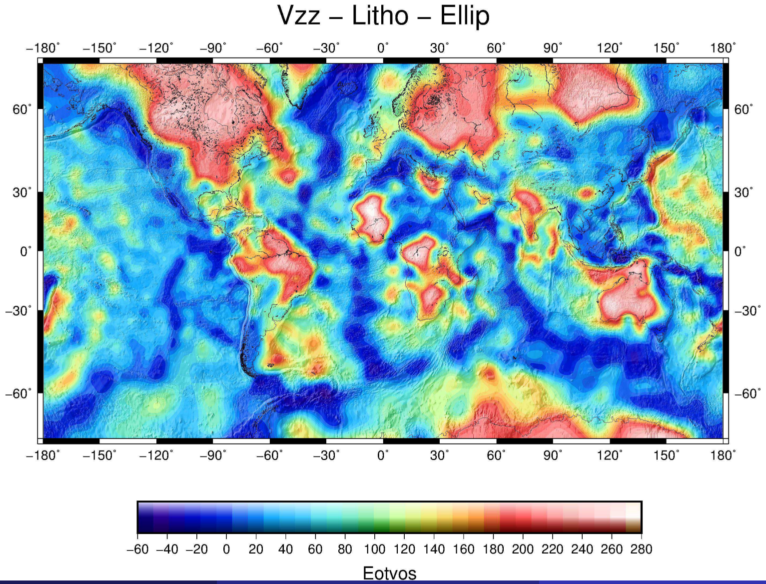
Conclusions

The numerical computation of global gravity gradient tensor up to the lithospheric mantle suitable with GOCE accuracy has to be performed with ellipsoidal prisms.

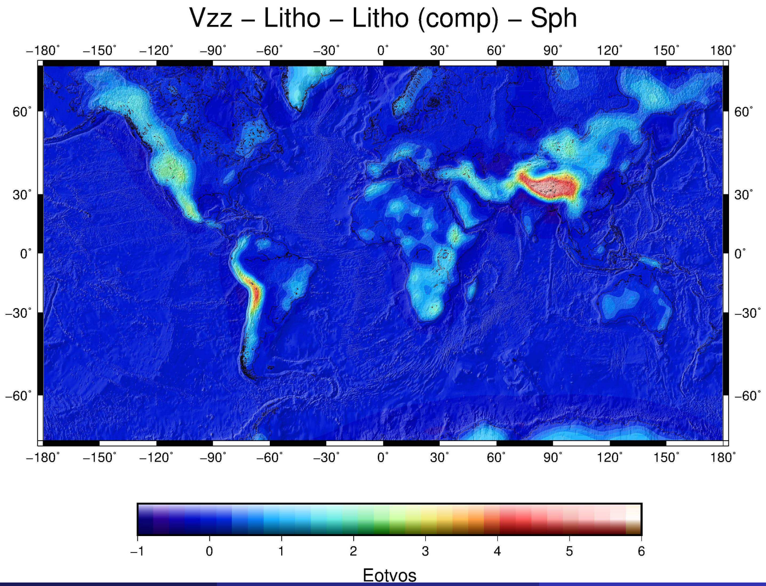
CRUST1.0 gravitational effect



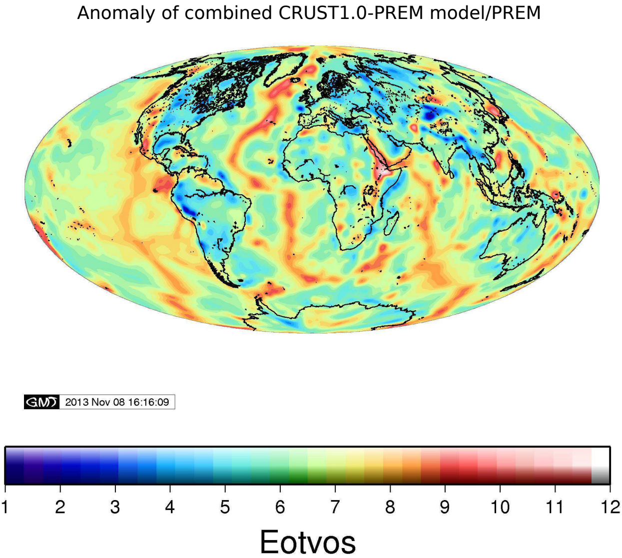
LITHO1.0 gravitational effect

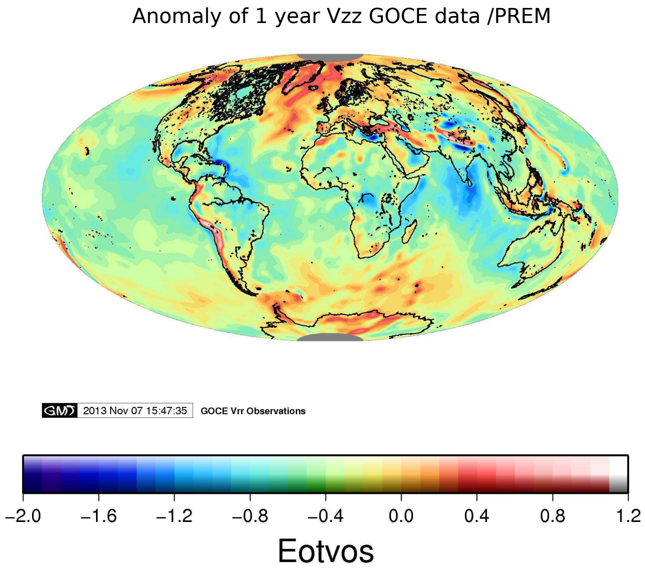


LITHO1.0 - LITHO1.0 after Airy isostatic compensation



Anomaly of CRUST1.0/PREM combined model with respect to PREM




Anomaly of 1 year Vzz GOCE data /PREM
GMT 2013 Nov 07 15:47:35
GOCE Vrr Observations
−2.0
−1.6
−1.2
−0.8
−0.4
0.0
0.4
0.8
1.2
Eotvos

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1. The need to use ellipsoidal prisms in the computation of global gravity effects produced by the whole lithosphere at 1 mE accuracy has been demonstrated conclusively.
2. The software using ellipsoidal prism to perform GLQ integration has been validated and is now efficient.

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2. Thanks to GOCE spectral sensitivity, suitable calculation for the purpose of comparison with geophysical models must necessarily include the lithosphere.

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1. Computation of the gravitational effect produced by LITHO1.0-PREM combined model.

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1. Computation of the gravitational effect produced by LITHO1.0-PREM combined model.
2. Computation of GOCE gravity gradient anomalies using 1 year GOCE data recently available as evenly sampled grids (GRD SPW 2)

Thank you for your kind attention!