The translational modes of the inner core : why they might not exist

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Strasbourg - 19^{th} november 2014

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Renaud Deguen, Thierry Alboussière
Fabien Dubuffet, Elise Poupart, Noé Rabaud

Jump conditions at phase changes

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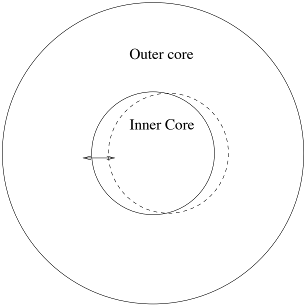
I. In which situation does the modified jump condition appears

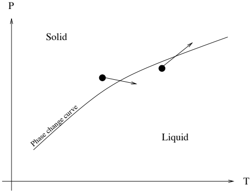
II. Highlight the problem with a very simple flow-model

III. A method to investigate boundary conditions

I. In which situation does the modified jump condition appears

1. The Slichter mode : an oscillatory mode





2. The permanent convective translation within the inner core

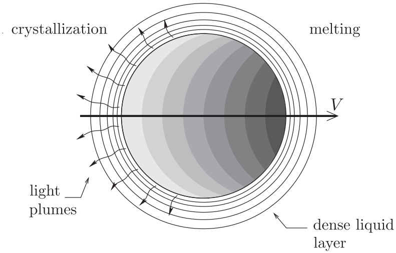
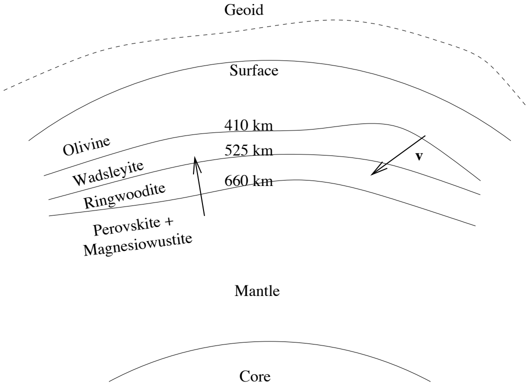


Figure 1. A schematic representation of the translation mode of the inner core, with the grey shading showing the potential temperature distribution (or equivalently the density perturbation) in a cross-section including the translation direction (adapted from Alboussière *et al.* 2010).

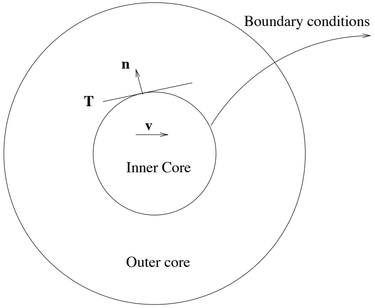
Figure: Deguen, 2013

Boundary condition ?

3. The mantel convection



Boundary condition ?



Usual boundary condition :

$$[\![\boldsymbol{\sigma}\boldsymbol{n}]\!] = 0$$

Notation : $[\![\boldsymbol{\sigma}\boldsymbol{n}]\!] := \boldsymbol{\sigma}^{+}\boldsymbol{n} - \boldsymbol{\sigma}^{-}\boldsymbol{n}$

Spherical geometry :

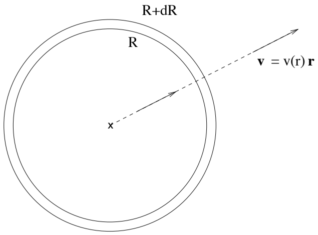
$$\sigma^{+}_{rr} = \sigma^{-}_{rr} \qquad \sigma^{+}_{r\theta} = \sigma^{-}_{r\theta} \qquad \sigma^{+}_{r\phi} = \sigma^{-}_{r\phi}.$$

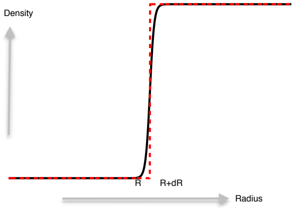
I will show that, at a permeable interface, like a phase change :

$$[\![\boldsymbol{\sigma}\boldsymbol{n}]\!] = \frac{2\gamma}{R}\,\boldsymbol{n} - \boldsymbol{\nabla}_{T}\gamma.$$

II. Highlight the problem :
a very simple flow-model (spherical)

A simple model to highlight the problem





Stokes Equation, viscous linear quasi-static fluid :

$$\nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\nabla \cdot \boldsymbol{\sigma} = 0,$$

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \eta \left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T \right) + \lambda (\nabla \cdot \boldsymbol{v}) \boldsymbol{I}.$$

Radial symetry :

$$\boldsymbol{v} = v(r)\boldsymbol{r},$$

$$\rho = \rho(r).$$

Not essential but simplifies the proof : $\eta = \text{constant}.$

A simple model to highlight the problem

$$\nabla \cdot (\rho \boldsymbol{v}) = 0,$$

$$\nabla \cdot \boldsymbol{\sigma} = 0,$$

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \eta \left(\nabla \boldsymbol{v} + (\nabla \boldsymbol{v})^T \right) + \lambda (\nabla \cdot \boldsymbol{v}) \boldsymbol{I}.$$

$$\partial_r (\rho v r^2) = 0$$

$$\partial_r \sigma_{rr} + \frac{1}{r}(2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\phi\phi}) = 0$$

$$\sigma_{rr} = -p + 2\eta \partial_r v + \lambda \partial_r (r^2 v)/r^2$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = -p + 2\eta v/r + \lambda \partial_r (r^2 v)/r^2$$

$$\partial_r \sigma_{rr} + 4\eta \partial_r \left(\frac{v}{r} \right) = 0.$$

Solution :

$$v = \frac{\mathrm{Cst}}{\rho r^2}$$

$$\sigma_{rr} + 4\eta \frac{v}{r} = \mathrm{Cst}'.$$

If $\rho \to$ discontinuous :

$$[\![\rho v]\!] = 0$$

$$[\![\sigma_{rr}]\!] = -\frac{4\eta}{R} [\![v]\!]$$

Young-Laplace law :

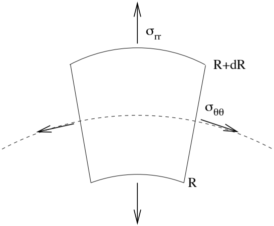
$$[\![\sigma_{rr}]\!] = \frac{2\gamma}{R}.$$

but with

$$\gamma = -2\eta [\![v]\!] = -2\eta \rho v [\![1/\rho]\!] .$$

The surface tension γ depends on the flow.

We try to understand why
Think of a thin balloon or a soap bubble



$$\sigma_{rr} = -p + 2\eta\, \partial_r v + \lambda \partial_r (r^2 v)/r^2$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = -p + 2\eta v/r + \lambda \partial_r (r^2 v)/r^2$$

Tectonics, deviatoric stress : push in

one direction $\leftrightarrow$ pull perpendicular

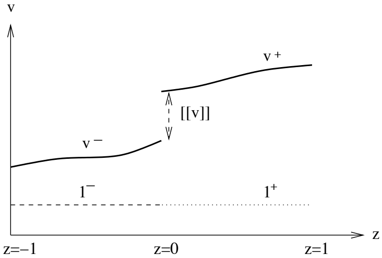
$$\sigma_{\theta\theta} = \sigma_{rr} - 2\eta\, (\partial_r v_r - v_r/r)\,.$$

$$\Longrightarrow \quad \sigma_{\theta\theta} \to \infty.$$

on the boundary.

Surface tension !

III. A method to investigate boundary conditions :
discontinuous functions (distributions)



Velocity written in a discontinuous form :

$$\boldsymbol{v} = \boldsymbol{v}^- \, \mathbb{1}^- + \boldsymbol{v}^+ \, \mathbb{1}^+,$$

i.e.

$$[\![\boldsymbol{v}]\!] = \boldsymbol{v}^+ - \boldsymbol{v}^- .$$

Then :

$$\boldsymbol{\nabla} \mathbb{1}^\pm = \pm \, \boldsymbol{n} \, \delta, \quad \boldsymbol{\nabla} \delta = \boldsymbol{n} \, \delta' .$$

and then

$$\boldsymbol{\nabla} \boldsymbol{v} = (\boldsymbol{\nabla} \boldsymbol{v})^- \, \mathbb{1}^- + (\boldsymbol{\nabla} \boldsymbol{v})^+ \, \mathbb{1}^+ + [\![\boldsymbol{v}]\!] \otimes \boldsymbol{n} \delta.$$

New interface conditions

General result, curved interface, discontinuous viscosity :

$$[\![\rho \boldsymbol{v} \cdot \boldsymbol{n}]\!] = 0 \qquad [\![\sigma \boldsymbol{n}]\!] = \frac{2\gamma}{R}\, n - \boldsymbol{\nabla}_T \gamma$$

with :

$$\gamma = -2\,[\![\nu]\!]\,\rho \boldsymbol{v} \cdot \boldsymbol{n} \quad \text{surface tension}$$

$$\frac{2}{R} = \boldsymbol{\nabla}_T \cdot \boldsymbol{n} = \text{total surface curvature}$$

$$\nu = \int \eta \partial_z \left(1/\rho\right)\, \mathrm{d}z \quad \text{the surface intrinsic property}$$

On a plane interface with a continuous viscosity :

$$\begin{array}{lllll} [\![\rho v_z]\!] & = & 0 & [\![\sigma_{zz}]\!] & = & 0 \\ [\![v_x]\!] & = & 0 & [\![\sigma_{xz}]\!] & = & \partial_x \left(2\eta\, [\![v_z]\!]\right) \end{array}$$

Numerical test

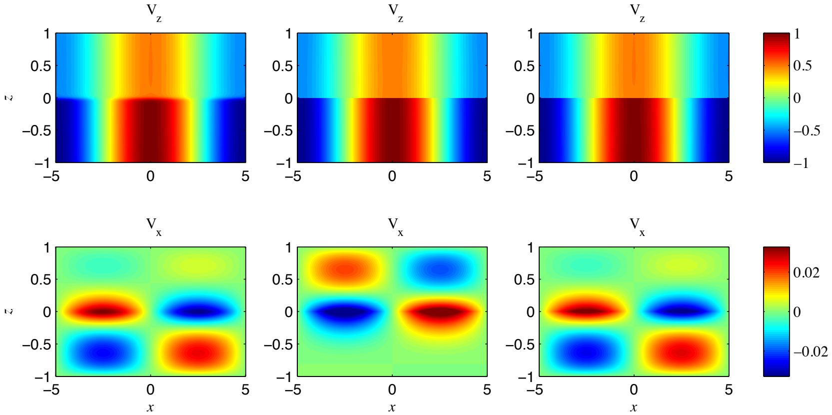
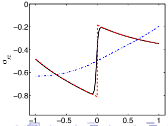


Figure: Velocities for the solutions as function of x and z. Left panels : numerical solution for a diffuse interface. Middle : numerical solution for a sharp interface by using the 'classical' jump conditions (continuous traction). Right : numerical solution for a sharp interface by using the 'classical' jump conditions (continuous traction).



Conclusions and prospect

Conclusion : when there is a mass transfert across a density jump interface, then the traction is not continuous. There is a 'dynamic' surface tension.

Prospect

- Inner-core convection
- Mantle convection (410 and 660 km phase changes)
- Rayleigh-Benard and Rayleigh-Taylor convection with phase change
- More generally fluid dynamics with phase change
- Slichter mode ?
- P-SV conversion ?
- (spherical) Shock waves ?
- Energy, temperature, entropy jumps ?
- Mesure of the surface tension ?

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Jump conditions and dynamic surface tension at permeable interfaces such as the inner core boundary

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