On the estimation of diffraction and verticality corrections in absolute gravimetry

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Introduction

FG5, FG5-X: most accurate absolute gravimeters based on laser interferometry, Standard uncertainty ≈ 2.5 μGal

INTERNATIONAL COMPARISONS – FG5s dominate

FG5s / AGs: 13/21 (2009), 17/21 (2011), 19/25 (2013)

Weights **FG5s / other AGs** : **> 4 / 1**

Reference gravity values are **strongly "FG5 dependent" !!!**

Systematic effects have to be captured:

FLOOR RECOIL

RESIDUAL AIR PRESSURE

ELECTRONICS

COLLIMATION

SELT ATTRACTION

TEST MASS ROTATION

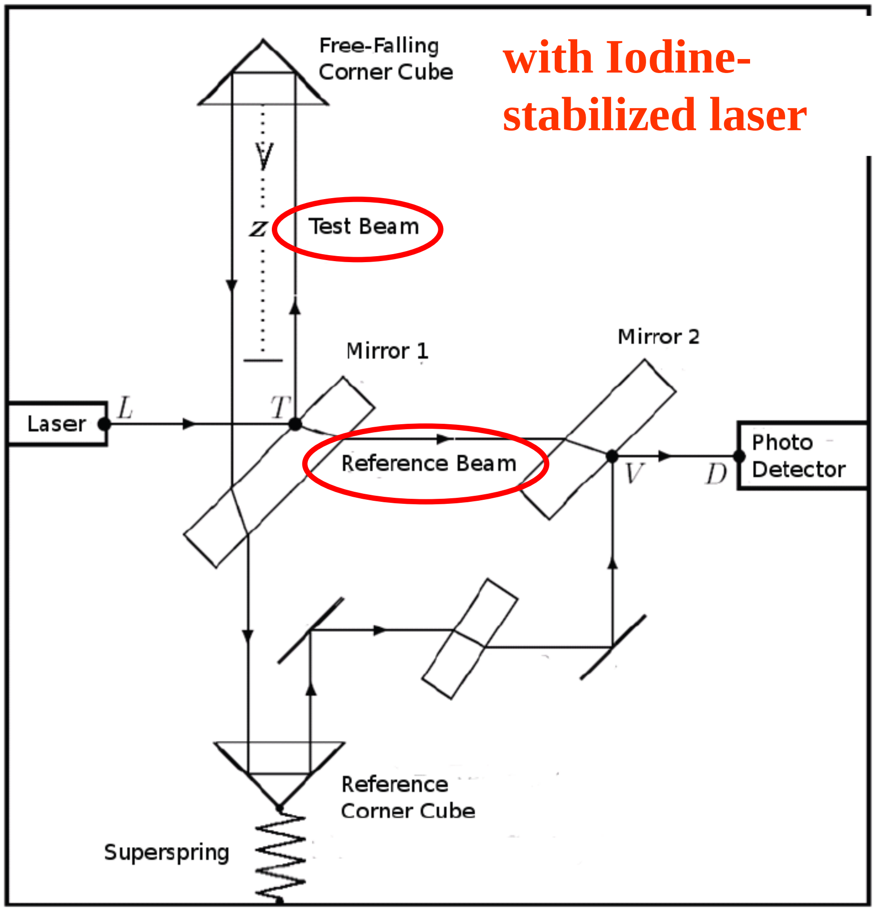
DIFFRACTION

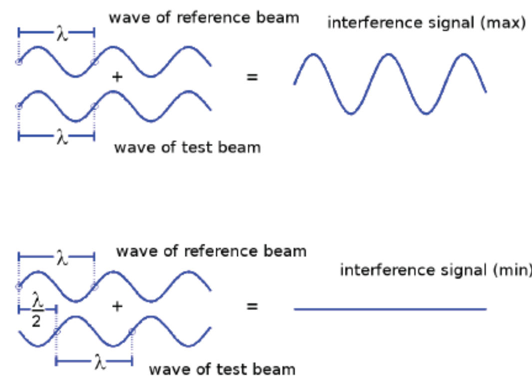
FINIT SPEED OF LIGHT

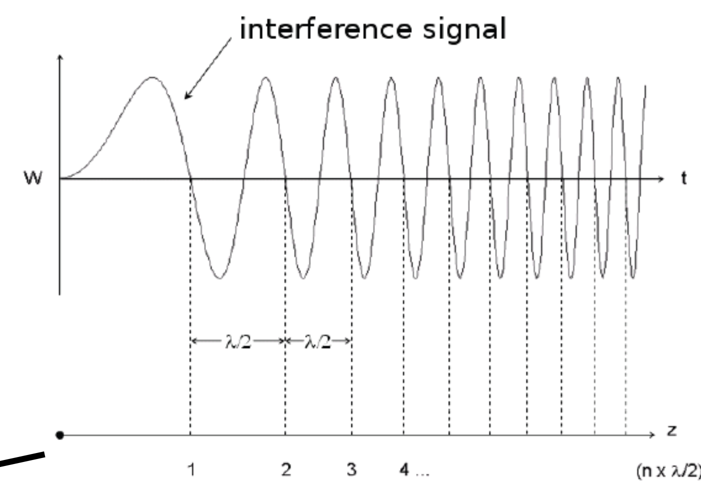
VERTICALITY

……..

FG5 Interferometer







Equally spaced data pairs (z_i, t_i)

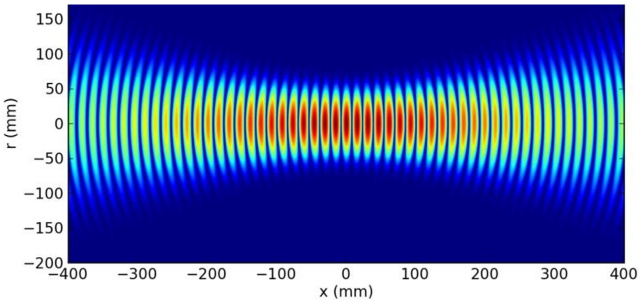
$$z_i = \frac{g_{ef}\, t_i^2}{2}$$

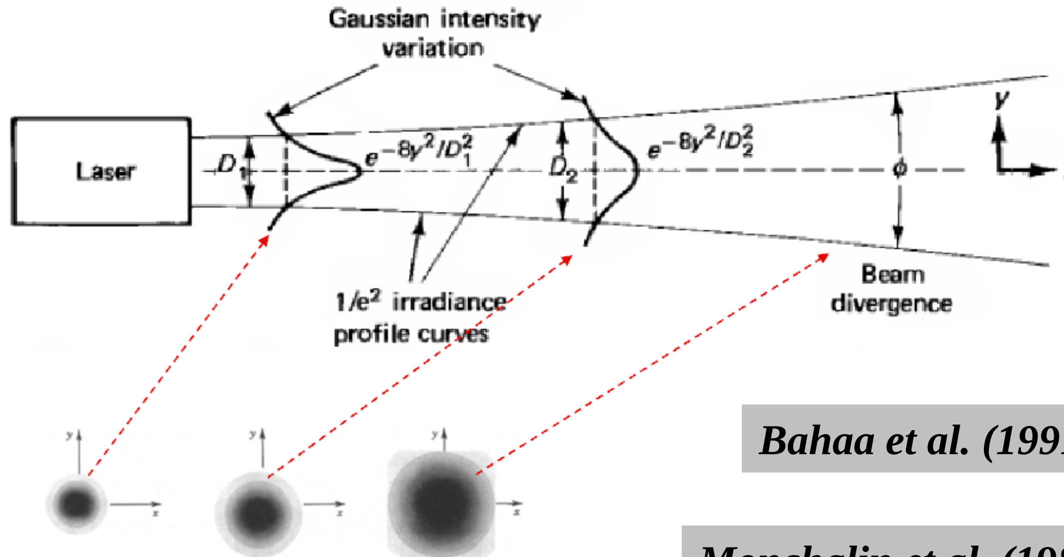
$$z_i = z_0 + v_0\,(t_i + \frac{\gamma}{6} t_i^3) + g_0(\frac{t_i^2}{2} + \frac{\gamma}{24} t_i^4)\,,$$

Diffraction effect

…caused by finite laser beam size

GAUSSIAN BEAM with curved wavefront

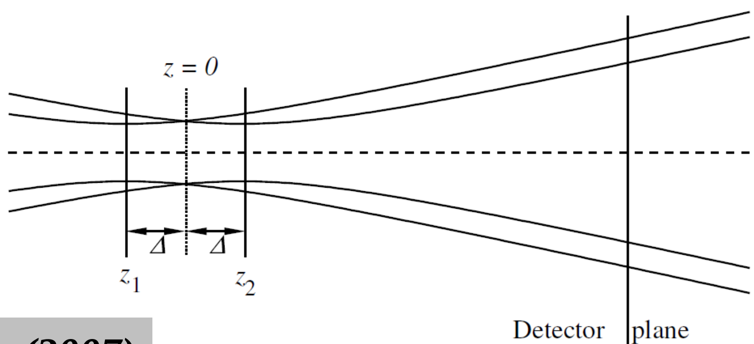




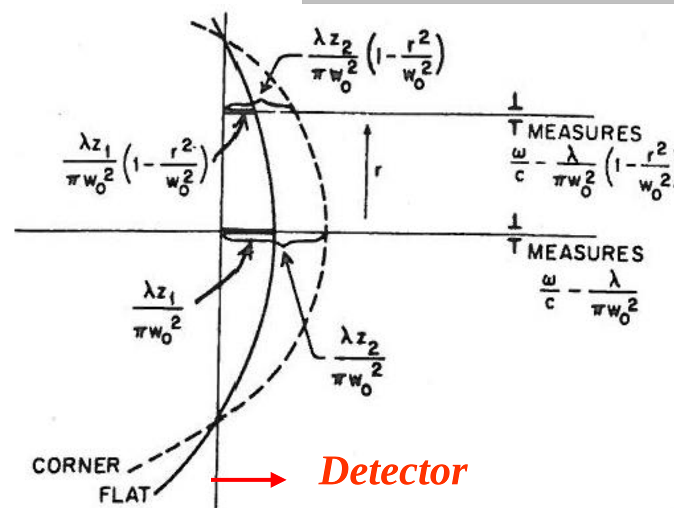
Bahaa et al. (1991)

Monchalin et al. (1981)

AGs use UNBALANCED interferometers



Robertsson (2007)

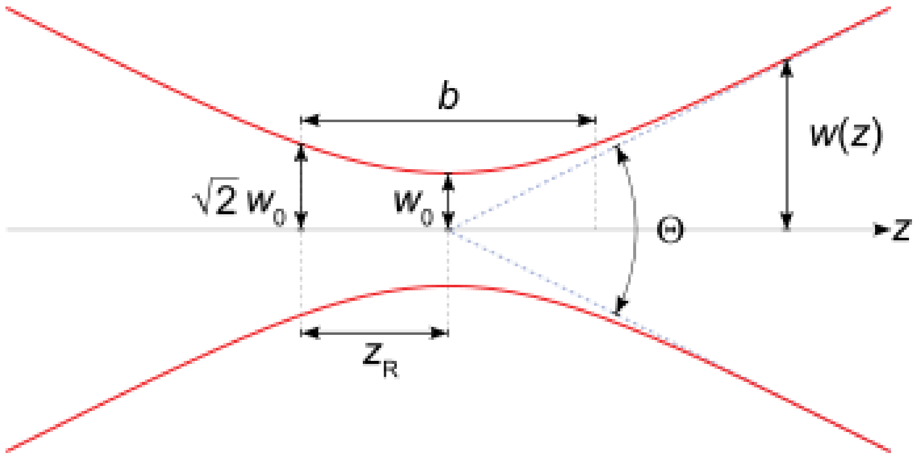


Diffraction effect

FG5s with red lasers (λ=633 nm):

Spot size (πw_0)≈ 5-10 mm ⇨ Waist (w_0)≈ 1.6-3.2 mm, Rayleigh range (z_R)=13-50 m

Reference and test beam: z_1≈ 1.2 m; z_2≈ 3.2 m ⇨ $z < z_R$ (NEAR FIELD)



The **beam waist w_0** of a laser beam is the location along the propagation direction where the beam radius has a minimum.

Van Westrum et al. (2003)

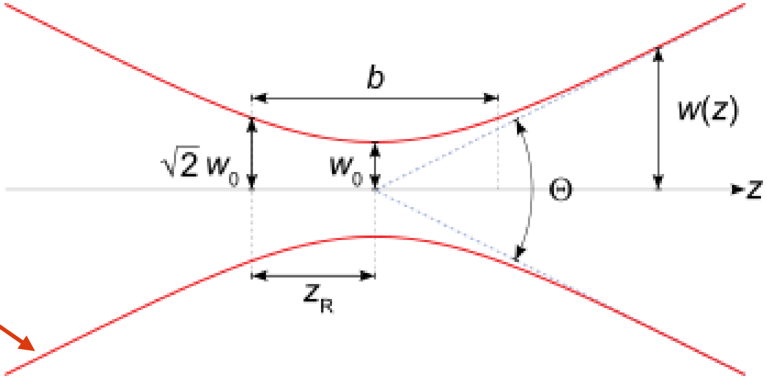
Robertsson (2007)

Diffraction correction (DC)

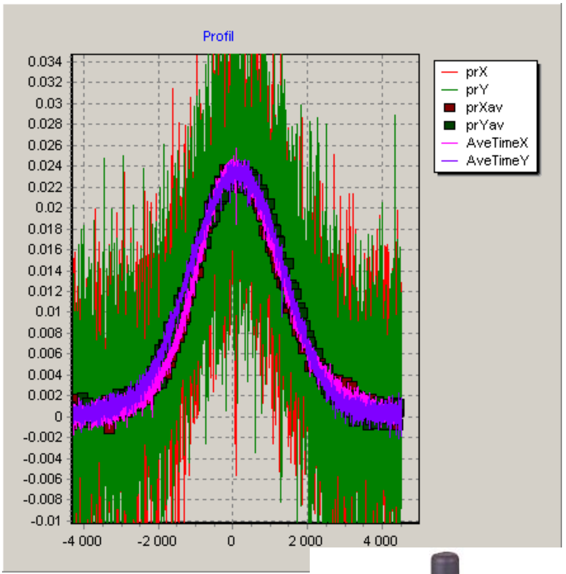
$$\Delta g = \frac{g\,\lambda^2}{4\pi^2 w_0^2} \Rightarrow 1.0-3.8\,\mu Gal$$

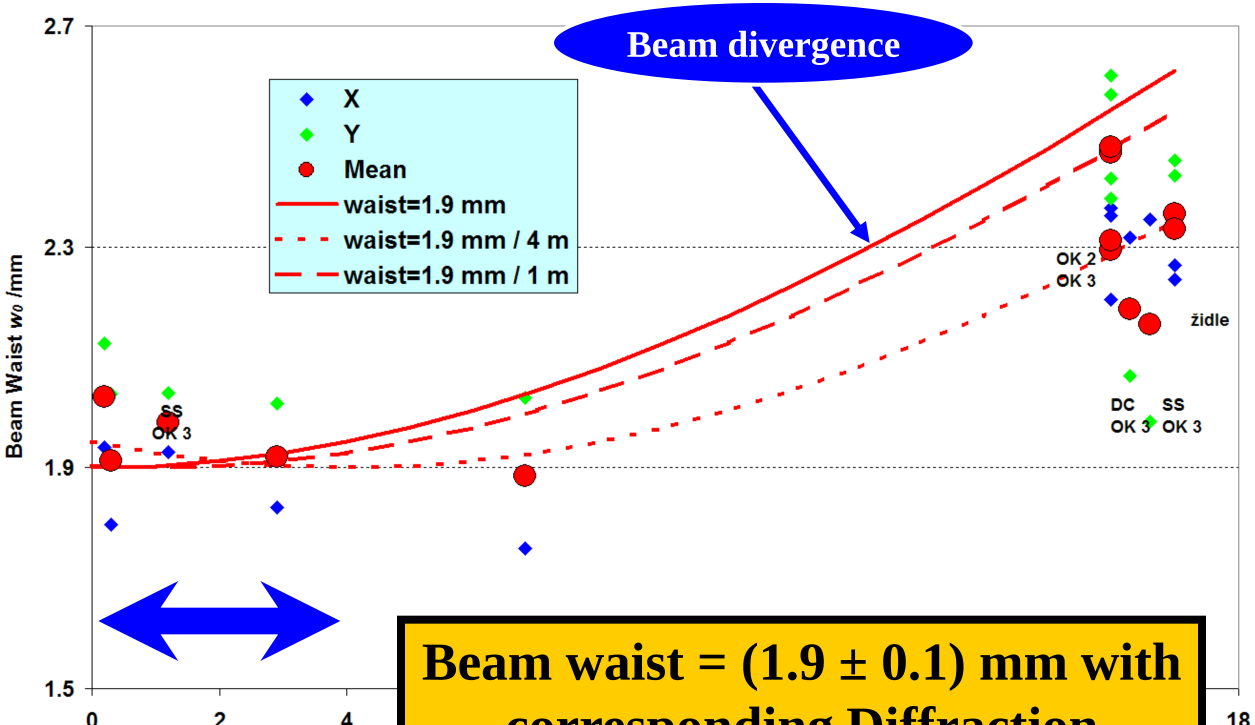
Usual DC applied by operators: 1.2 – 1.6 μGal

Beam waist determination



$1/e^2$ intensity profile ⇒ 86.4% laser energy is inside

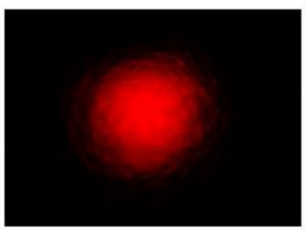
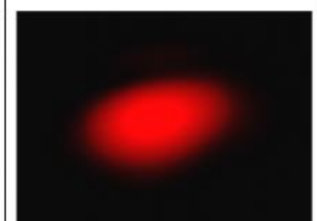




FG5 related distances

Beam waist = (1.9 ± 0.1) mm with corresponding Diffraction Correction of (2.8 ± 0.3) μGal

Beam quality

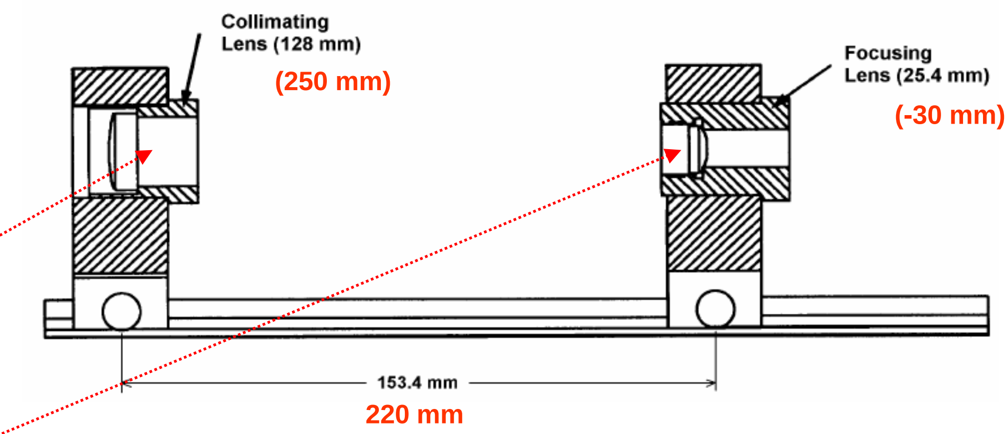
Distance [m]	0.5 m	4 m	8 m	12 m	16 m
Test Beam after beam splitting					
Test Beam from Dropping Chamber					
Test Beam from Supersping					

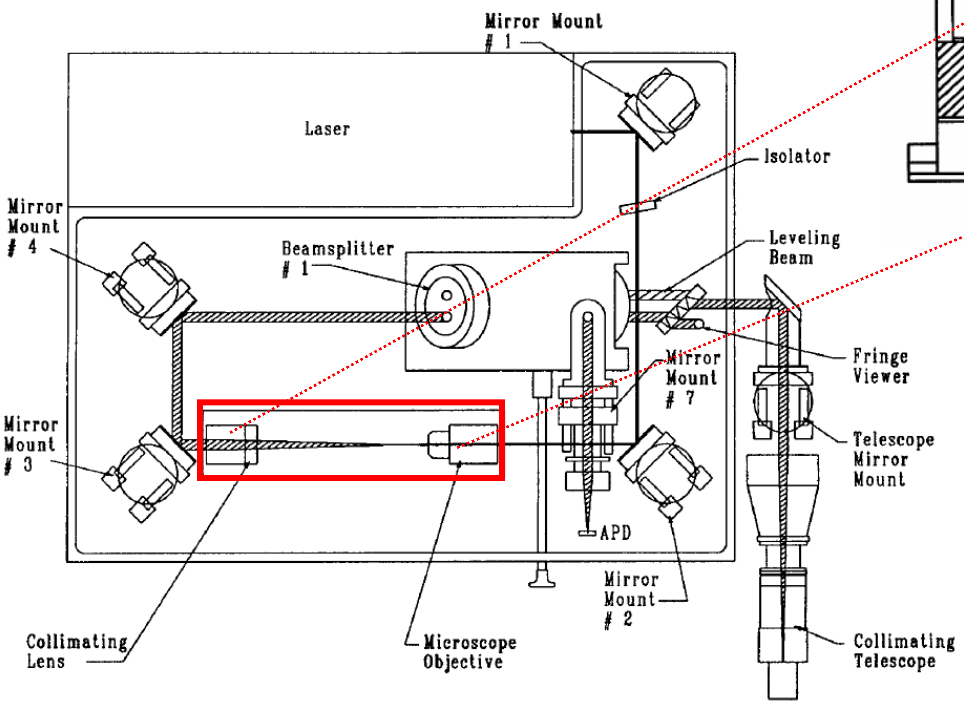
Beam waist verification (**1.9 ± 0.2) mm**.

Significant beam quality degradation (**wavefront distortion**) when it **passing the Superspring**

DC validation by an experiment

Two pairs of focusing-collimating **lenses** were used in the laser interferometer to reach different **beam waists of (1.9±0.1) mm and (3.1±0.1) mm** with corresponding **DC** of **(2.8±0.3) μGal and (1.0±0.1) μGal**, respectively.





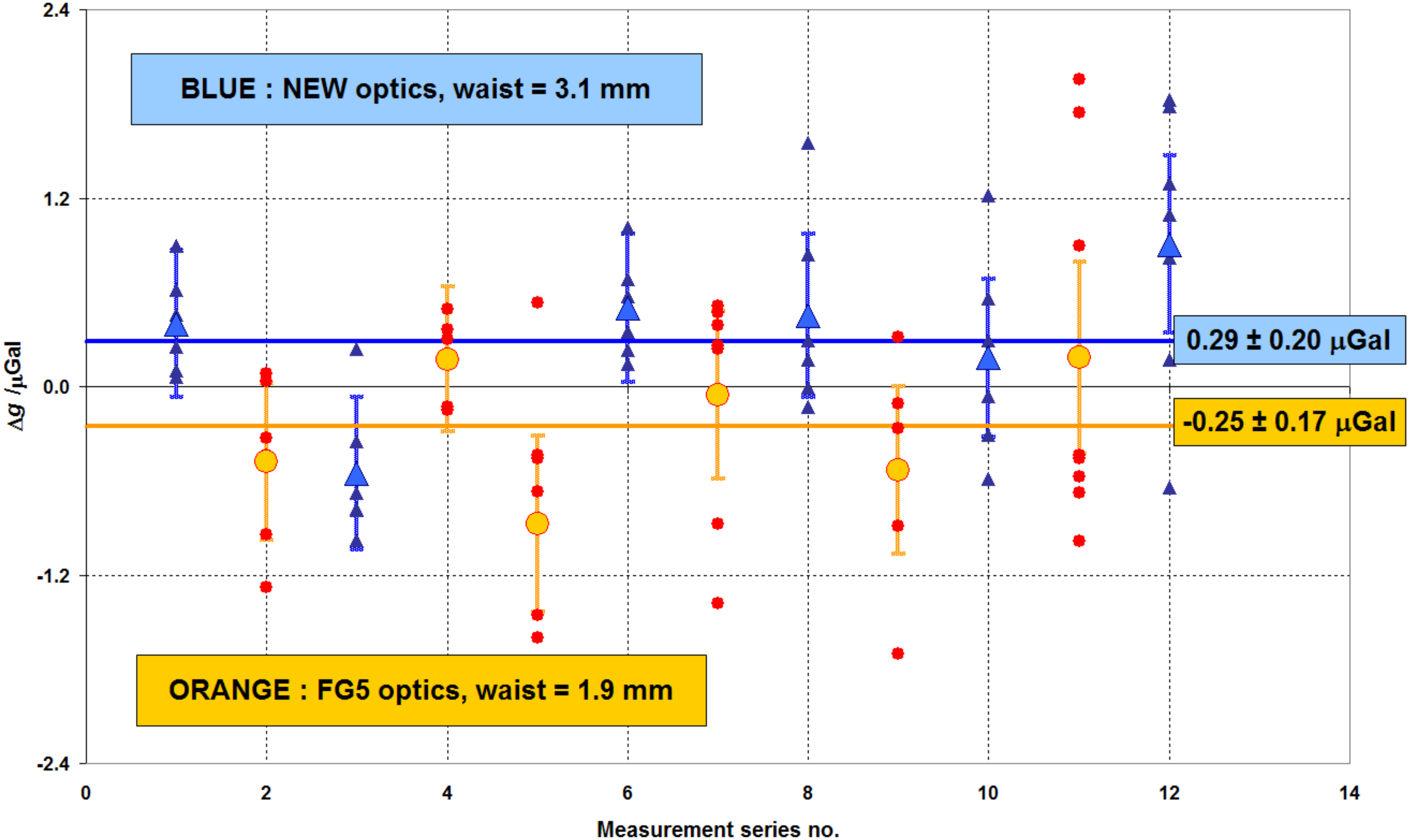
Theoretical difference of

(1.7 ± 0.4) μGal

has been checked by means of precise absolute gravity measurements.

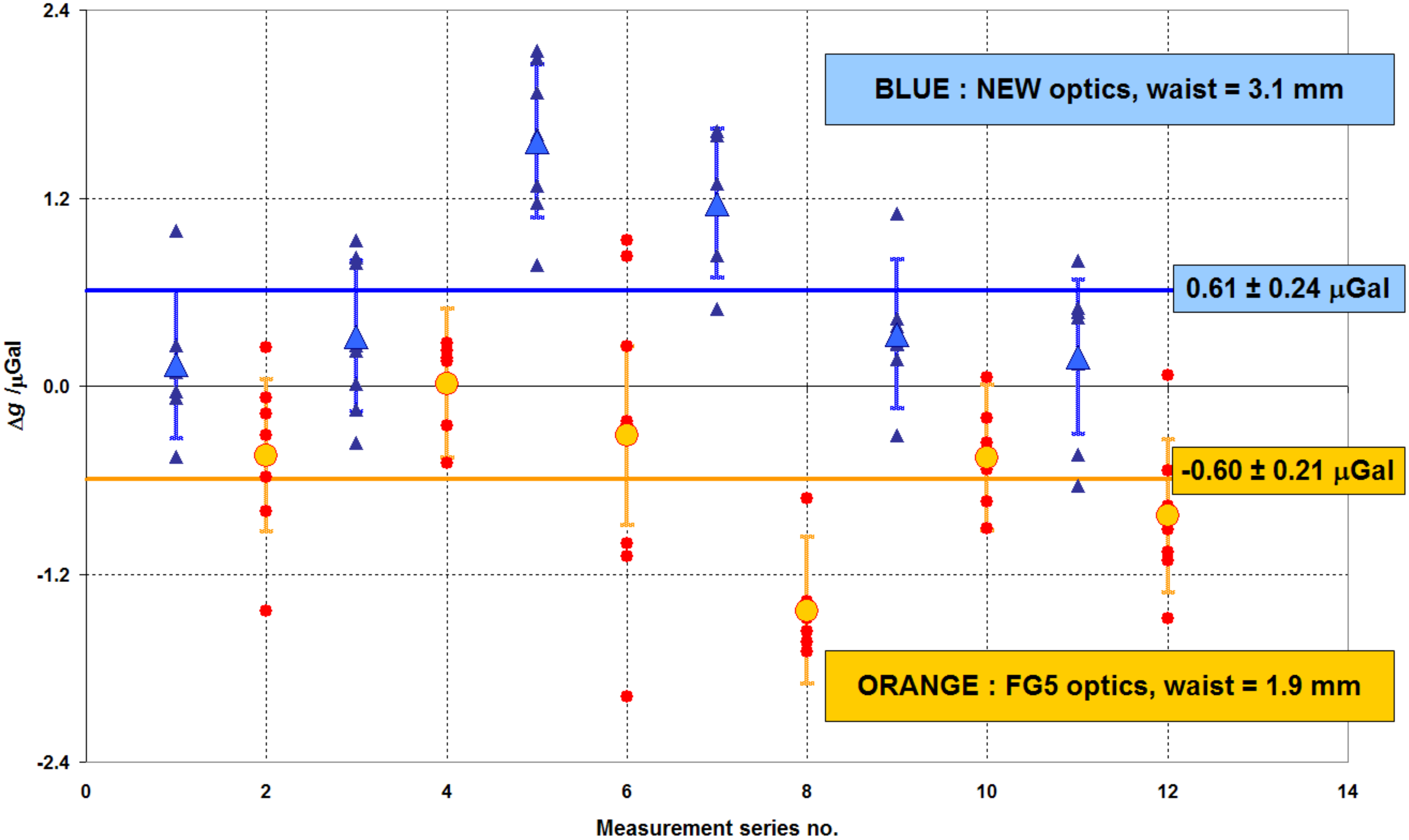
AG experiment (1)

November 2013, 12 series with at least 1000 drops, σ < 0.2 μGal, repeatability ≈ 0.5 μGal



AG experiment (2)

May 2014, 12 series with at least 1000 drops, σ < 0.2 μGal, repeatability ≈ 0.5 μGal



Results

"New-FG5" optics; spot size 10 mm vs. 6 mm

Theoretical difference …………….. **(1.72 ± 0.32) μGal**

First validation **(0.54 ± 0.26) μGal**

Δ = 0.67 ± 0.41 μGal

Second validation **(1.21 ± 0.32) μGal**

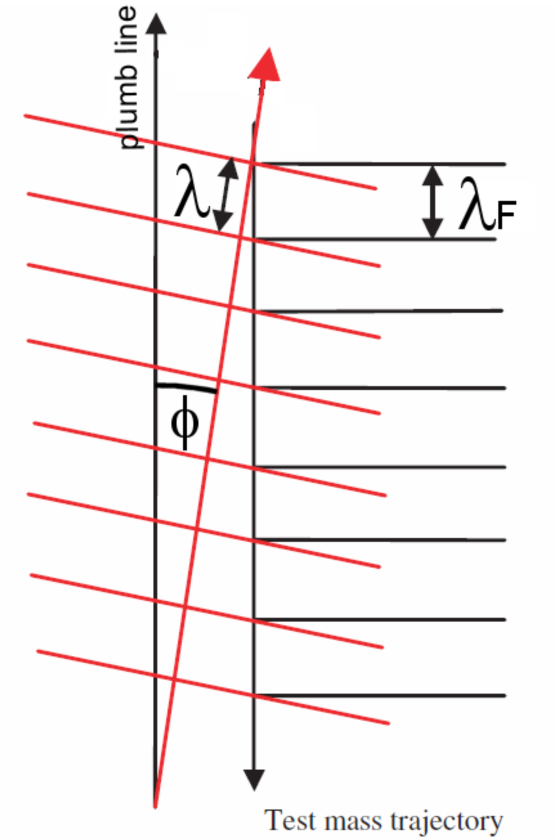
Δ = 0.91 ± 0.38 μGal

Experimental difference ……… …. **(0.81 ± 0. 20) μGal**

The experiment showed on the **measurable difference** of **0.8 ± 0.2 μGal** for a typical **spot sizes of** FG5s ranging from **6 mm to 10 mm**. However, the theoretical difference should reach **1.7 ± 0.3 μGal (for an ideal Gaussian beam)**.

Other possibility: the diffraction correction is simply overestimated. Si crystal lattice measurement for new definition of the kilogram also shows on the overestimation of the diffraction correction.

Verticality correction



If the laser is not aligned along the plumb line, the test mass falls distance of $\lambda_F = \lambda / \cos \varphi$ (longer than the measured distance λ). Therefore, the **measured gravity** g_m becomes **lower** than the **"real"** $\boldsymbol{g}$.

$$g_m = g \cos\phi \approx g\left(1 - \frac{\phi^2}{2}\right)$$

The correction is always positive

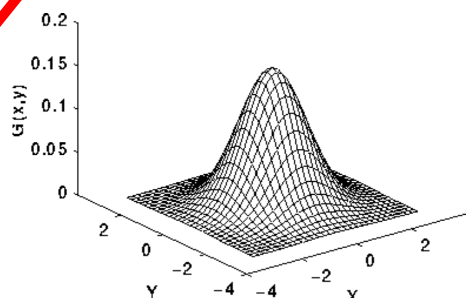
$$\Delta g_{Ve} = g\,\phi^2 / 2$$

The mean correction will depend on the probability density function of ϕ

$$\phi = \sqrt{x^2 + y^2}$$

In case x, y have normal distribution N(0, σ_x), N(0, σ_y), and $\sigma = \sigma_x = \sigma_y$ then

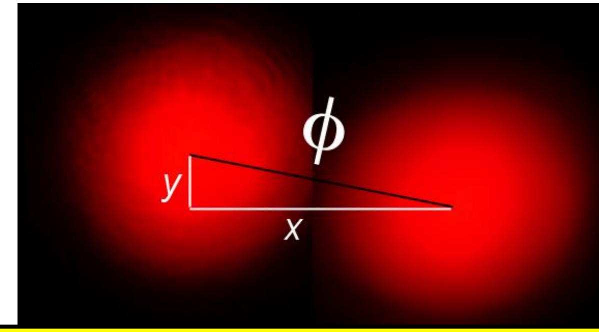
$$f(\phi) = f(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



… and the correction itself has Chi-squared distribution

Verticality check

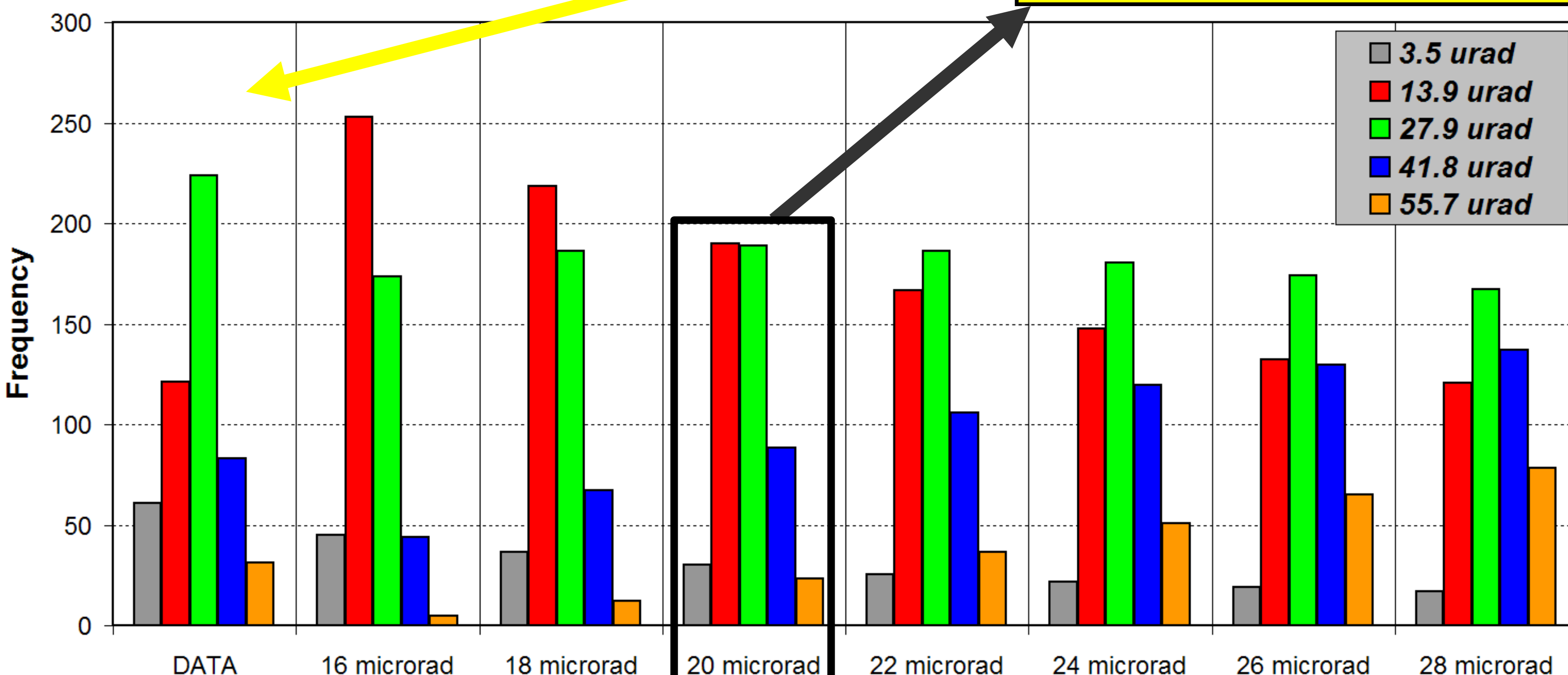
Real deviations used for statistics: regular verticality control of the FG5#215 during measurements



ϕ /µrad	Δg_{Ve} /µGal	**Frequency**
3.5	0.01	**61**
13.9	0.10	**121**
27.9	0.38	**224**
41.8	0.86	**83**
55.7	1.52	**31**

Best agreement: $\sigma_x = \sigma_y = 20$ µrad

Δg_{Ve} **= +0.4 µGal**



Summary and conclusions

We have estimated the **beam waist radius of (1.9 ± 0.1)** mm with corresponding **theoretical** diffraction correction **of (2.8 ± 0.3) μGal** which significantly **differs** from the value of **1.2 μGal typically used** for FG5s**.**

Experimental results with two pairs of optical lenses proved that we can apply **different diffraction corrections for FG5s** depending on the beam size. However, it looks that other effects (beam quality, Gouy shift, methods of measurement), may significantly influence (up to 2 μGal) the diffraction correction.

Results of the FG5#215 have to be corrected for **+ 0.4 μGal** to compensate for the one-side systematic error due to deviations from the verticality. Statistical approach have been found for the evaluation of the correction. Mean of the correction is represented by the corresponding mean of the **Chi-squared distribution.**

Thank you for your attention!

NTIS

Technologická agentura
České republiky

This work was supported by the European Regional Development Fund (ERDF), project "NTIS - New Technologies for Information Society", European Centre of Excellence, CZ.1.05/1.1.00/02.0090 and the project TAČR/2619/2012 of the TAČR.