Comparison of the global gravitational effects obtained respectively from LITHO1.0 geophysical model and GOCE satellite data

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GOCE ⇒ full tensor gravity gradients = direct, global, high spatial resolution (∼ 100 km), 1 mE accuracy
LITHO1.0
Available at 1°x1° resolution

## LITHO1.0: physical parameters

<table>
<thead>
<tr>
<th>num</th>
<th>layer</th>
<th>parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ice</td>
<td>(thickness), (vp), (vs), (ρ), (Q)</td>
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<tr>
<td>2</td>
<td>water</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
<td>sediment layer 2</td>
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<tr>
<td>5</td>
<td>sediment layer 3</td>
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</tr>
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<td>6</td>
<td>upper crust</td>
<td>thickness, vp, vs, ρ, (Q)</td>
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<tr>
<td>7</td>
<td>middle crust</td>
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<tr>
<td>8</td>
<td>lower crust</td>
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</tr>
<tr>
<td>9</td>
<td>lithospheric mantle (lid)</td>
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*Parameters unmodified from the starting model are shown in parentheses.*

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**Starting models**

**CRUST1.0**  
Laske et al., 2012

**LLNL-G3D**  
Simmons et al., 2012  
Pasyanos, 2010 (thickness)

Kennett et al., 1995
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Possible comparison: FTG components deduced respectively from forward modelling and GOCE data.

Starting models

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Outline

1. Numerical computation of Earth gravity gradients
2. Overview of first results
3. Work in progress and future work
Brief review of GOCE reference frames

Gradiometer Reference Frame (GRF)

GOCE Level 2 Product Data Handbook, EGG-C, 2014

Accelerometer Reference Frames (ARF)

less sensitive axis
Brief review of GOCE reference frames

Local North Oriented Frame

Original figure from: GOCE Level 2 Product Data Handbook, EGG-C, 2014
Computation of gravity gradients in the LNOF

LITHO1.0 model versus GOCE data

November 18th 2014
Possible by means of numerical integration given a geophysical model
Computation of gravity gradients in the LNOF

Possible by means of numerical integration given a geophysical model

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Gravitational potential:  
\[ V = \int \int \int_{\text{Terre}} \frac{G \rho(M) d\tau_M}{MP}, \]

\[ G = 6.67 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \]
Basic notations for the gravity field

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2. Gravitational acceleration:

\[ \mathbf{g} = \nabla P(V) = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}, \quad V_\alpha = \partial_\alpha V, \quad \alpha = x, y, z \]
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   Symmetric tensor with null trace out of masses.
Spherical vs ellipsoidal prism
Spherical vs ellipsoidal prism

- Two mass elements for decomposing the Earth body
Spherical vs ellipsoidal prism

- Two mass elements for decomposing the Earth body
- Ellipsoidal prisms are well-suited for modelling the flattening of the Earth
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- Two mass elements for decomposing the Earth body
- Ellipsoidal prisms are well-suited for modelling the flattening of the Earth

Spherical prism

Ellipsoidal prism
Gauss-Legendre Quadrature

Prism domain

\[ \int_{\lambda_1}^{\lambda_2} \int_{\varphi_1}^{\varphi_2} \int_{u_1}^{u_2} f(\lambda_P, \theta_P, r_P, \lambda_S, \varphi_S, u_S) \, du_S \, d\varphi_S \, d\lambda_S \]

Any gravity related quantity

Observation point (spherical coordinates)

Source point (ellipsoidal coordinates)
Gauss-Legendre Quadrature

Prism domain

\[ \int_{\lambda_1}^{\lambda_2} \int_{\varphi_1}^{\varphi_2} \int_{u_1}^{u_2} f(\lambda_P, \theta_P, r_P, \lambda_S, \varphi_S, u_S) du_S d\varphi_S d\lambda_S \]

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Source point (ellipsoidal coordinates)

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Gauss-Legendre Quadrature

\[ \int_{\lambda_1}^{\lambda_2} \int_{\varphi_1}^{\varphi_2} \int_{u_1}^{u_2} f(\lambda_P, \theta_P, r_P, \lambda_S, \varphi_S, u_S) du_S d\varphi_S d\lambda_S \]

Prism domain

Any gravity related quantity

Observation point (spherical coordinates)
Source point (ellipsoidal coordinates)

\[ \sum_{i=1}^{n_\lambda} \sum_{j=1}^{n_\varphi} \sum_{k=1}^{n_u} \omega_i \omega_j \omega_k f(\lambda_P, \theta_P, r_P, \lambda_{S_i}, \varphi_{S_j}, u_{S_k}) \]

Prism domain

Quantity to be integrated

Observation point (spherical coordinates)
Source point at Gauss-Legendre nodes (ellipsoidal coordinates)
Gauss-Legendre weighting coefficients
Gauss-Quadrature validation: example for $V_{zz}$

$e_{T_{rr}} = e_{V_{zz}} = (V_{zz})_{GLQ} - (V_{zz})_{exact \, formulas}$ for an ellipsoidal layer of constant density.
Gauss-Quadrature validation: example for $V_{zz}$

$e_{Tr\gamma} = e_{V_{zz}} = (V_{zz})_{GLQ} - (V_{zz})_{exact\ formulas}$ for an ellipsoidal layer of constant density.

Roussel et al., 2014
To what extent the spherical approximation remains valid?
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PREM layers

Crust + upper mantle

Lower mantle

Outer core

Inner core

Roussel et al., 2014
To what extent the spherical approximation remains valid?

**PREM layers**

- Ocean
- Crust
- Lid
- Low Velocity Zone
- Transition zone
- Lower mantle

GOCE expected accuracy

Roussel et al., 2014
Conclusions

The numerical computation of global gravity gradient tensor up to the lithospheric mantle suitable with GOCE accuracy has to be performed with ellipsoidal prisms.
CRUST1.0 gravitational effect

Vzz – Crust – Ellip

Eotvos

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Anomaly of CRUST1.0/PREM combined model with respect to PREM

Anomaly of combined CRUST1.0-PREM model/PREM
Anomaly of GOCE 1 year $V_{zz}$ data with respect to PREM

Anomaly of 1 year Vzz GOCE data /PREM
The need to use ellipsoidal prisms in the computation of global gravity effects produced by the whole lithosphere at 1 mE accuracy has been demonstrated conclusively.
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The software using ellipsoidal prism to perform GLQ integration has been validated and is now efficient.
Conclusions and prospects
In our experiment, the direct computation of gravity gradients by interpolation of available GOCE data leads to underestimated gravity anomaly values; further investigation must be carried out.
Conclusions and prospects

1. In our experiment, the direct computation of gravity gradients by interpolation of available GOCE data leads to underestimated gravity anomaly values; further investigation must be carried out.

2. Thanks to GOCE spectral sensitivity, suitable calculation for the purpose of comparison with geophysical models must necessarily include the lithosphere.
Conclusions and prospects
Computation of the gravitational effect produced by LITHO1.0-PREM combined model.
Conclusions and prospects

1. Computation of the gravitational effect produced by LITHO1.0-PREM combined model.
2. Computation of GOCE gravity gradient anomalies using 1 year GOCE data recently available as evenly sampled grids (GRD SPW 2)
Thank you for your kind attention!