

# Quantifying the evolution of static elastic properties as crystalline rock approaches failure

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## Abstract

The pervasive damage of rock by dilational microcracks strongly influences rock strength and macroscopic elastic properties. In this study, two types of experiment were performed to investigate and quantify the contribution of microcracking to the static elastic response of Westerly granite as it approaches failure. They are (1) increasing-amplitude cyclic loading experiments and (2) constant-amplitude cyclic loading experiments, some of which incorporated a ‘load-hold’ component to explore the elastic response of time-dependent effects such as stress corrosion. We report values for the tangent moduli in the region of the stress–strain curve where the values are the same for an unloading cycle as they are for the subsequent loading cycle. Hence, although there is hysteresis between these two curves, we assume perfect elasticity in this region. This approach allows the evolution of static elastic properties as rock approaches failure to be documented in great detail, in contrast to previous work that has generally been limited to the linear elastic region of the stress–strain curve. Increasing-amplitude stress cycling causes a gradual reduction in sample stiffness, equating to a decrease in Young’s modulus (~11%) and an increase in Poisson’s ratio (~43%) measured at a constant stress level. Elastic properties are also seen to have a strong stress-dependency during loading (~46% increase in Young’s modulus from 20 to 100 MPa). Experiments devised to promote time-dependent microcracking had a negligible contribution to the evolution of static elastic properties over the timeframe and dry conditions under which our experiments were conducted. Such results can be applied to our understanding of the mechanics, stress distribution and fault displacement models within and surrounding fault zones.

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## 1. Introduction

Elasticity is a fundamental physical property of rock. It controls the distribution and magnitude of sub-yield stress within the Earth’s crust, the propagation of elastic (seismic) waves through rocks and is used to relate strain measurements to in situ stresses present in the crust. Distributed damage and imperfections affect the rock strength and elastic properties and are ubiquitous in materials, including rocks under crustal stress conditions [1,2]. Stressing brittle rock leads to the further development of distributed damage [3], which is manifest

as dilational microcracks and can eventually lead to brittle failure [4,5].

Research on the topic has generally been biased towards the effect of ‘damage’ on the ultimate failure strength (damage mechanics), and less attention has been directed towards the effect of microcracking on the elastic properties of rocks. However, the evolution of elastic properties as rock approaches failure has many important geological applications. Among these applications is the understanding of stress surrounding geological faults. Faults have been shown to have a high microcrack density near the fault core that decreases exponentially with distance [6–10]. Faulkner et al. [11] showed that these microcracks change the elastic properties of rock and hence the stress state surrounding faults. Hence a change in elasticity for rocks in fault zones has implications for the mechanics, stress distribution and fault displacement models within and

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surrounding fault zones. These effects are not limited to fault zones, and increased microcrack density in any part of the brittle crust is likely to result in a change in the state of stress if the elastic properties are modified by crack density.

As an illustration of the problems encountered we provide the following examples. Both linear elastic and non-linear elastic (or post-yield) crack models predict a linear relationship between rupture displacement during an earthquake or total fault displacement ( $u$ ) and either rupture length or total fault length ( $L$ ); however, attempts to find universal-scaling laws for  $L/u$  have generally failed [12]. Some have suggested that these models have partly failed because they do not take into account the changes in mechanical properties, in particular Young's modulus as the fault zone evolves [13]. This was supported observations made by Peltzer et al. [14] where interferometric synthetic aperture radar observations of the 1997 Manyi earthquake in Tibet infer non-linear elastic behaviour of the crust that is dependent on the stress applied (compressive or tensile). Non-linear elastic crack models generally invoke some region of yield around the crack or rupture tip, and this is often interpreted as a cloud of microcracks [15]. Although in these computations the yield strength of the rock is specified and microcracking or yield is expected at approximately 50% of the yield stress [8], the accompanying changes in the elastic properties are not accounted for. Stresses and displacements around underground excavations are also often calculated assuming linear elasticity. This leads to erroneous predictions of the deformation and of the initiation and extent of failure around underground excavations [16]. Brown et al. [16] report that using a slightly modified theory that incorporates elastic moduli that change with confining pressure, results are more consistent with aspects of those obtained in hydrostatic compression tests. Lau and Chandler [17] have also highlighted the use of elastic properties in the design of deep geologic repositories. The characterization of elastic moduli evolution as rock approaches failure can provide further improvement to such calculations.

Several experimental studies have shown that elastic properties [17–21] depend on the extent of damage within a rock. Theoretical approaches to characterizing rock elastic properties have also been used. Lyakhovskiy et al. [22] use models to predict changes in the effective elastic modulus against axial strain, focusing on the change as loading reverses from uniaxial compression to uniaxial tension. However one of the principal shortcomings of previous work has been that the elastic properties are only calculated in the linear elastic region and hence valuable information on the elastic properties at stresses closer to failure (in the non-linear elastic region) are not given [e.g. 18,21]. Typically the linear elastic region ranges only up to ~40% of the failure stress and consequently there is a great deal of information lost. In other work, the 'elastic properties' are quoted at stresses above the linear elastic region, but these values are simply the local gradient of the stress–strain curve (the 'tangent' moduli), or the ratio of

the principal strains (for Poisson's ratio), without any attempt to establish if the non-linear portion of the stress strain curve is reproducible and hence elastic. Indeed, in many cases, the quoted values for 'Poisson's ratio' are in excess of 0.5, indicating inelasticity [17,19].

Time-dependent brittle deformation is a fundamental and ongoing process in the Earth's upper crust, and leads to the failure of rock at relatively low stresses. Static fatigue or 'brittle creep' tests used to study this process have generally only considered the long-term strength of crystalline rock [19,23–28], but have yet to explore its potential influence on the development of crack networks leading to changes in the elastic properties.

The change in elastic properties during deformation can also be monitored by inversion of the ultrasonic wave velocities measured in experiments, given the density and assuming isotropy [29–39]. However, it has long been recognized that there are differences in the static and dynamic elastic properties of rocks [40–44]. This is due to the amplitude and frequency of the stress that is applied to the rock, and grain scale response to these applied stresses—such as intrinsic inelasticity. As Ciccotti and Mulargia [43] note: 'the static response of rocks to large-scale stresses can be quite different from that estimated by the seismic measurements and should be investigated further'.

In light of all this previous work, we conclude that the characterization of static elastic property development for fractured rock approaching failure is a fundamental, yet poorly understood phenomenon. In this work we aim to measure elastic properties outside the linear elastic region and hence quantify elastic property development at stresses closer to sample failure. Our study quantifies the change in static elastic properties as a product of large amplitude stress cycles using a new methodology to determine the elastic properties in the non-linear elastic part of the stress strain curve. It allows the tangent elastic moduli to be determined for specific values of applied stress. We also perform 'load-hold' tests in order to try to identify any time-dependent processes that may be operative and their effect on the elastic properties. The paper first introduces background and previous work on rock elastic properties. We then explain experimental technique and how the data were interpreted to obtain static elastic properties. Finally, we present results from the tests and briefly discuss these results in light of the likely micromechanical processes that may be operative.

## 2. Experimental procedure

In this study, two types of experiment were performed to investigate the contribution of microcracking to the resultant strain damage and static elastic response of rock approaching failure. They are (1) increasing-amplitude cyclic loading experiments and (2) constant-amplitude cyclic loading experiments, some of which incorporated a 'load-hold' component. All experiments were performed on

oven-dried (at 80 °C, see [45]), 1 in diameter cylindrical samples of initially intact Westerly granite and conducted using an unconfined ( $\sigma_2$  and  $\sigma_3 = 0$ ), uniaxial compression apparatus. Westerly granite (from south-east Rhode Island, USA) has been extensively tested in rock mechanics due to its small grain size, low initial crack density and that it is generally considered to be isotropic [27,46 and references therein]. Hence it can yield a high level of experiment reproducibility under carefully controlled test conditions.

It is pertinent to note however that Hadley [47,48] and Scholz and Koczyński [49] described Westerly granite as exhibiting mild dilatancy anisotropy; also, granite is usually inherently anisotropic in its mechanical behaviour due to the existence of preferentially oriented microcracks [50,51, and references therein]. The experimentally induced microcracking will encourage an anisotropic fabric within the sample [34,52], resulting in anisotropic elastic property development as deformation progresses. In our study, as in many other previous experimental studies involving Westerly granite, we assume isotropy within the sample, and if the change in elastic parameters in directions other than the principal strain axes is required, then further, more elaborate experiments would be required.

The samples were cored from the same block, in the same direction (see Fig. 1 for core orientation) to allow results to be comparable even with the existence of a mild anisotropy fabric, cut so their length-to-diameter ratios were in excess of 2.5 [53] and precision-ground to strict tolerances ( $\pm 0.02$  mm) as to the squareness of their edges. Steel spacers, of the same diameter as the specimen, were used at each end of the sample and held in place by plastic tubing to help reduce the problem of ‘end effects’ [53]

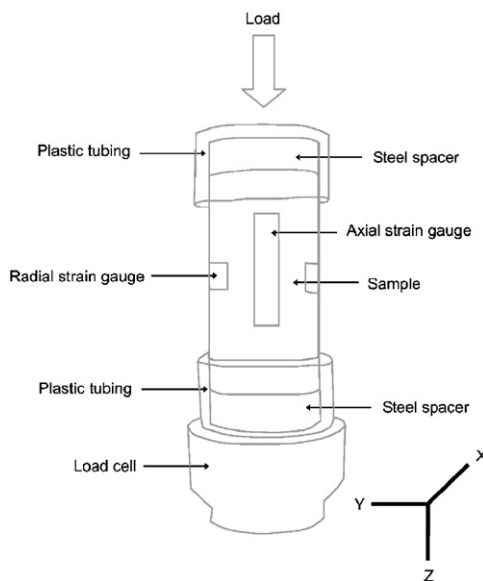


Fig. 1. Schematic diagram showing sample set-up during experimentation. Core orientation is also indicated for use in conjunction with thin section photomicrographs. Z-direction is along the sample axis.

(Fig. 1). All of the above factors are fundamental for the production of a viable data set [54].

Strain gauges in an axial and circumferential configuration were wired into a full Wheatstone bridge circuit, which utilized a dummy sample, and were used to monitor the development of the principal strains (Fig. 1). All strains, and stress from the load cell, were continuously recorded during experimentation. The stress was increased and decreased by small increments ( $\sim 2$  MPa) during experiments via a hand-controlled valve and the load-rate was kept at approximately 2.5 MPa/min. The unconfined compressive strength (UCS) of the cored Westerly granite samples was determined to be  $\sim 200$  MPa (from samples WG-01 and WG-02). All experiments were carried out under ambient laboratory conditions, with temperatures  $\sim 20$  °C. Test conditions for all experiments are summarized in Table 1.

Many researchers have investigated the stress–strain behaviour of brittle rock under compression. The failure process can be broken down into a number of stages (Fig. 2) characterized by changes in the axial and circumferential strain response recorded during uniaxial compression tests [19,21,55–57]. The stages include: OA, the region where the stress–strain response is concave upwards; AB, the region where the stress–strain behaviour is very nearly linear; BC, the region where the stress–strain response is concave downwards; and after C, post-peak behaviour, dependent on the stiffness of the testing machine, can be seen following the peak stress.

The elastic properties of materials are generally obtained from region AB of the curve, where the linear elastic constants can be determined. However, as long as there is a reproducible relationship between stress and strain such as for a perfectly elastic material, the relationship need not necessarily be linear. Perfect elasticity implies that, during loading and unloading, the same path is followed. Hence the energy stored in the specimen during loading will be released during unloading. In this case there are no unique values for the elastic moduli; instead there will be a range of values corresponding to the level of stress on the specimen. These elastic parameters can be obtained by the local gradient of the stress–strain curve (termed the ‘tangent’ modulus in [55]).

In the case of rocks, however, perfectly elastic behaviour does not strictly occur, as once the yield point is exceeded there is hysteresis, implying that some of the energy stored during loading has been dissipated into the specimen. In low temperature experiments, this energy is generally assumed to be used to create new crack surface area. In order that we might obtain elastic parameters in the non-linear region of the stress strain curve, a test for perfect elasticity, or something approximating to it, is required.

To this end, we analysed each set of unloading and subsequent loading curves for our stress-cycle experiments (Fig. 3a) and adequately fitted both curves with third-order polynomial relationship in order to capture the true form of the stress–strain curve with two inflexion points. These

Table 1  
Experimental conditions. UCS-unconfined compression strength, DNF-did not fail

Sample	Experiment type	Cycle amplitude	Cycles to failure	Load-hold stress	Load-hold time (hrs)	Number of hold periods	Failure stress (MPa)	Notes
WG-1	UCS	N/A	1	N/A	N/A	N/A	190	
WG-2	UCS	N/A	1	N/A	N/A	N/A	205	
WG-3	Increasing amplitude	Small	10	N/A	N/A	N/A	195	Cycle amplitude too small in WG-03 and 04
WG-4	Increasing amplitude	Small	10	N/A	N/A	N/A	193	
WG-5	Increasing amplitude	Large	0	N/A	N/A	N/A	133	Test failed
WG-6	Increasing amplitude	Large	7	N/A	N/A	N/A	169	
WG-7	Constant amplitude	N/A	N/A	0.7UCS	1	0	DNF	Test failed
WG-8	Increasing amplitude	Large	0	N/A	N/A <td N/A	196		
WG-9	Constant amplitude	N/A	N/A	0.7UCS	1	6	DNF	
WG-10	Constant amplitude	N/A	N/A	0.8UCS	1	6	DNF	
WG-11	Constant amplitude	N/A	N/A	0.8UCS	0.5	6	DNF	
WG-12	Constant amplitude	N/A	N/A	0.8UCS	0.75	6	DNF	
WG-13	Constant amplitude	N/A	N/A	0.8UCS	6	6	DNF	

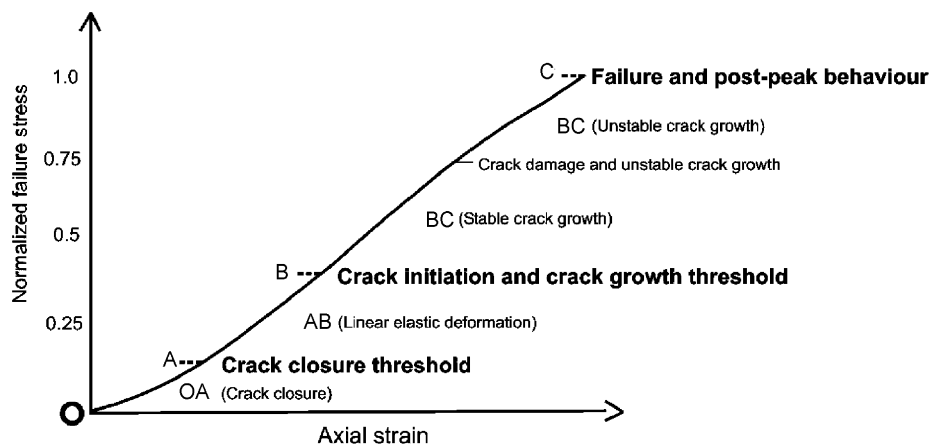


Fig. 2. Schematic diagram showing the general shape of the stress–strain curve in a typical experiment. Also shown are the approximate positions of the stages of crack development in brittle rock under uniaxial compression [55–57].

fits can be differentiated so that tangent modulus can be calculated for any given stress. The tangent modulus was calculated for both of the curves at 10 MPa and then at every 10 MPa increment thereafter. Perfect elasticity has been assumed where the gradients of unloading and the subsequent loading curves are approximately equal (Fig. 3b) despite the hysteresis. We contend that by comparing an unloading curve with a subsequent loading curve, no additional microcrack damage will be imparted to the specimen until the previous highest stress on the loading cycle is exceeded. This assertion is supported by acoustic emission analysis made in triaxial tests conducted

by other workers during stress cycling and is termed the Kaiser effect (see reviews by Holcomb [58], Lockner [59] and Lavrov [60]). However, in our experiments AE events were not recorded; in some stress-cycling studies AE have been recorded [52,61,62]. If the gradients are equal, we assume that the same micromechanical processes are operating both during unloading and subsequent loading. Hence, although hysteresis maybe present, if the same tangent modulus is reproducible during both unloading and loading, we have assumed these values provide a reasonable approximation of the perfect elastic properties (and are hence termed ‘Young’s modulus’ in our reported

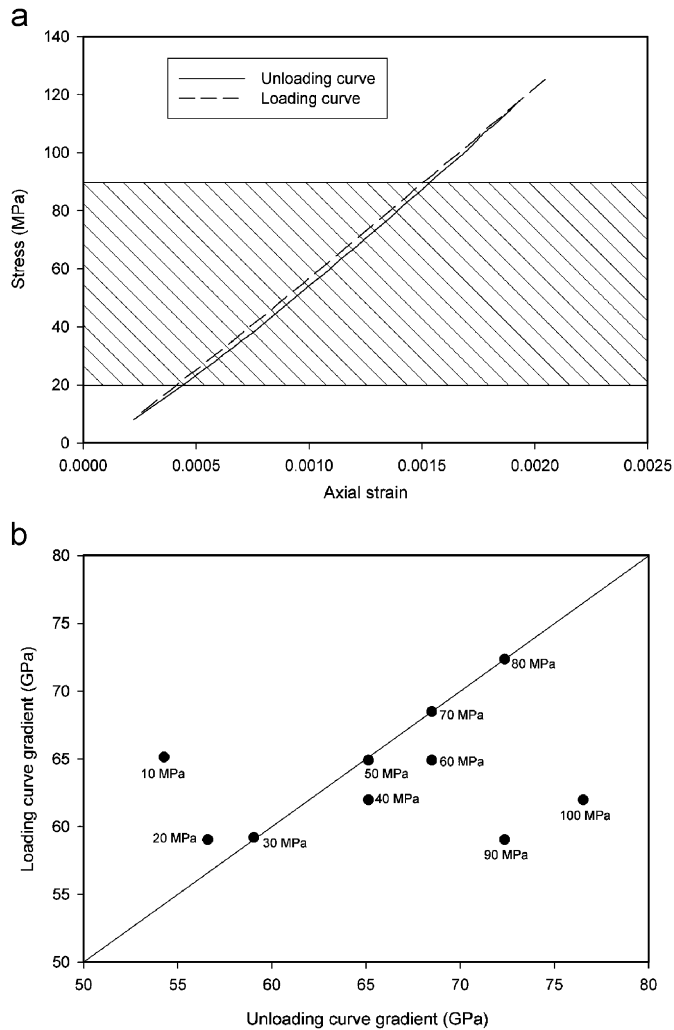


Fig. 3. (a) Example of one of the sets of unloading and subsequent loading curves taken from sample WG-08. Hatched area denotes area where elasticity has been confirmed using the approach detailed in the text. Outside of the hatched area, the data were not used to determine elastic properties. (b) Plot showing the calculated unloading and subsequent loading curve gradients (Young's moduli) for the set of curves in (a). Values were excluded from the dataset if they deviated by more than 5 GPa from the line of equal gradient (in this case 10, 90 and 100 MPa).

data). Our data, obtained in this way, show that the Poisson's ratio does not extend above 0.5 and as such suggests the validity of our approach. This method has allowed us to greatly extend the region in which the elastic properties can be obtained.

### 3. Experimental results

#### 3.1. Increasing-amplitude cyclic experiments

In order to try to investigate the evolution of static elastic properties as rock approaches failure samples of Westerly granite were subjected to increasing-amplitude cyclic loading experiments. The stress-peak of the large-amplitude cycles started at  $\sim 80$  MPa ( $\sim 0.4$ UCS) and was increased by  $\sim 12$  MPa after each completed cycle until the

sample failed. The low starting value ensured that an acceptable number of cycles were completed before rock failure and so that the changes in static elastic properties as the sample approached failure could be recorded in detail. Fig. 4 shows a representative stress–strain curve during increasing-amplitude cyclic loading for Westerly granite (sample WG-08). The graph shows that permanent strain has accumulated during experimentation and therefore crack damage mechanisms must have been operative. The approximate locations of the stages of stress–strain behaviour for brittle material [55–57] are indicated (as in Fig. 2). Their positions are termed ‘approximate’ since Eberhardt et al. [63] found that they are more accurately determined using an approach that combines AE event counts with a moving point regression analysis. Brace et al. [56] defined the position of crack initiation (stage B) to be 0.3–0.5 UCS and Bieniawski [57] suggested that unstable crack growth is likely to occur between 0.7–0.85 UCS. The boundary between the crack closure threshold stage (stage A) and the linear elastic deformation region (region AB) was simply defined by the change to a linear gradient (Fig. 2).

From these data, and using our test for ‘perfect’ elasticity, the Young's modulus and Poisson's ratio were calculated (from sample WG-08) and their evolution plotted with respect to stress (Fig. 5a) and number of cycles (Fig. 5b). Both graphs demonstrate that large-amplitude stress cycling results in an overall reduction in sample stiffness. Fig. 5a shows that elastic properties exhibit a strong stress-dependency. If a single stress cycle is considered in Fig. 5a (cycle 7 in this example), Young's modulus is seen to increase rapidly (total increase of  $\sim 46\%$  from 20 to 100 MPa), whereas Poisson's ratio demonstrates only a very modest decrease with respect to increasing stress, until the last few cycles when a larger decrease is observed. Both elastic properties are seen to change at relatively constant rates. If a single stress value is considered in Fig. 5b (50 MPa in this example), the Young's modulus is seen to decrease ( $\sim 11\%$ ) and the Poisson's ratio increase ( $\sim 43\%$ ) as the number of cycles is increased and before the sample reaches failure. Young's modulus is seen to decrease at a relatively constant rate; however, Poisson's ratio is seen to increase exponentially just before failure. It must be noted that number of cycles is not an independent variable since their amplitude increased with each successive cycle. Mechanisms operating during the increasing-amplitude experiments have large effect on the evolution of static elastic properties.

#### 3.2. Constant-amplitude cyclic experiments

Samples of Westerly granite were also subjected to constant-amplitude cyclic experiments. In such experiments, time-dependent effects such as stress corrosion were encouraged by implementing ‘load-hold’ periods between cycles so that its influence on static elastic properties could be investigated. Cycles were required to evaluate the effect

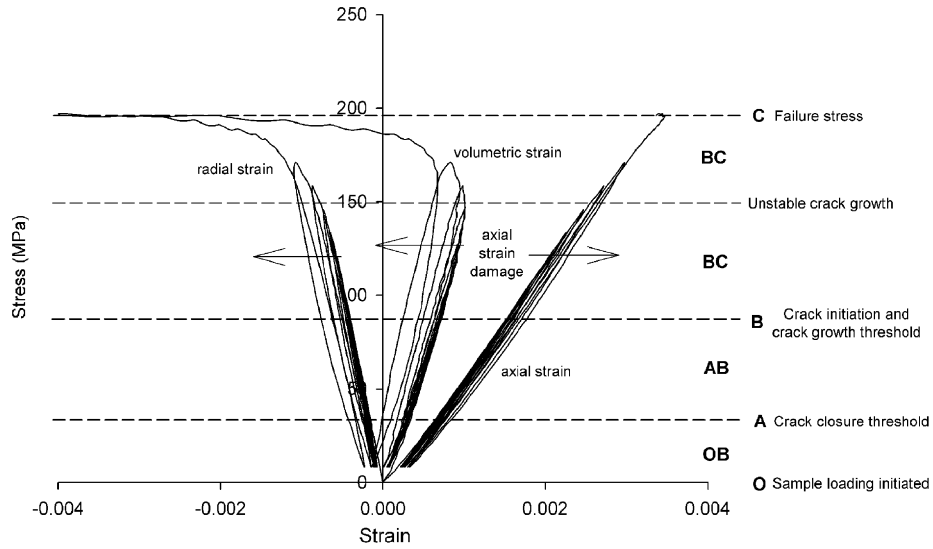


Fig. 4. Representative stress–strain curve for Westerly granite under increasing-amplitude cyclic loading (sample WG-08). Stress–strain stages have also been included as in Fig. 2 [55–57]. Arrows indicate how the axial, circumferential and volumetric strains change during the accumulation of microcrack damage during the experiment.

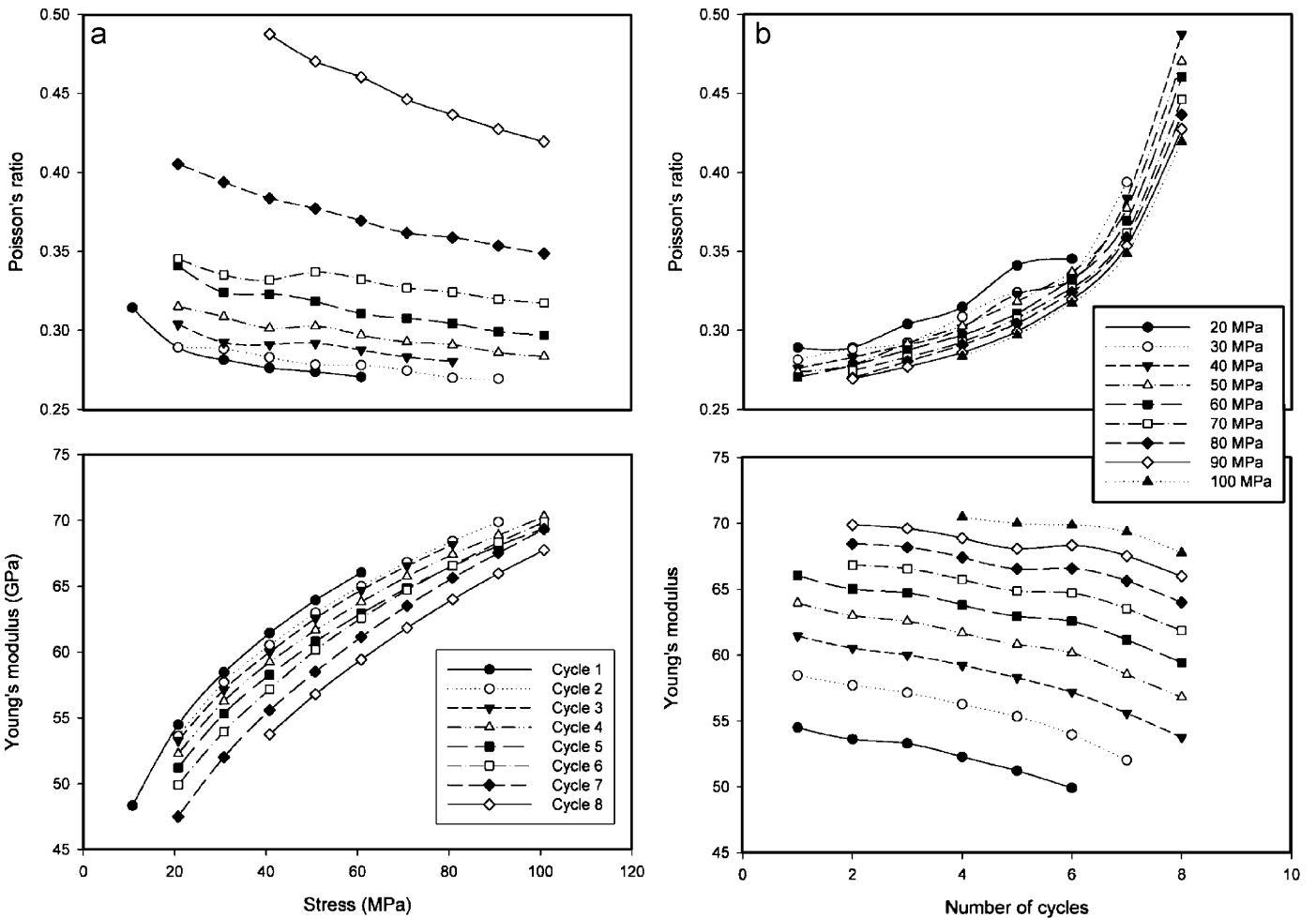


Fig. 5. (a) Elastic property development in Westerly granite with increasing stress during increasing-amplitude cyclic experiments (sample WG-08). (b) Elastic property development in Westerly granite with increasing number of cycles during increasing-amplitude cyclic experiments (sample WG-08).

of stress corrosion on static elastic property development since stress–strain curves are necessary for their calculation. The experiments were performed at two different proportions of the unconfined compressive strength (%UCS) and load-hold durations between cycles were also varied between experiments (see Table 1 for test conditions). Experiments were carried out at load-hold axial stresses of either 0.7 UCS or 0.8 UCS—both within the unstable crack propagation region as defined by Bieniawski [57]. High stresses at the load-hold periods were selected in an attempt to ensure the effect of stress corrosion can be seen within laboratory timescales. The UCS was assumed using the average failure stress from two conventional UCS tests (samples WG-01 and WG-02). We note that as crack damage accumulates in the sample during experimentation then the UCS will decrease, therefore the %UCS used in the hold periods is expected to have increased slightly during testing. Fig. 6 shows a representative stress–strain curve for Westerly granite during a constant-amplitude experiment (sample WG-10). The graph shows that only a small amount of permanent strain accumulated during experimentation and therefore crack damage mechanisms must have been operating, albeit at a lesser extent compared to increasing-amplitude experiments.

Using the methodology outlined previously, the evolution of the elastic properties has been calculated with respect to number of cycles for sample WG-10 (Fig. 7). It shows that crack damage mechanisms cause a gradual reduction in sample stiffness with respect to number of cycles, equating to a small decrease in Young’s modulus (~1%) and a more substantial increase in Poisson’s ratio (~13%). Both are seen to change at a relatively constant rate with respect to number of cycles. Decreasing the %UCS from 0.8 to 0.7 UCS the percentage decrease in Young’s modulus remains approximately the same; however, the increase in Poisson’s ratio reduces to ~6%. However, the data from just the load-hold periods show

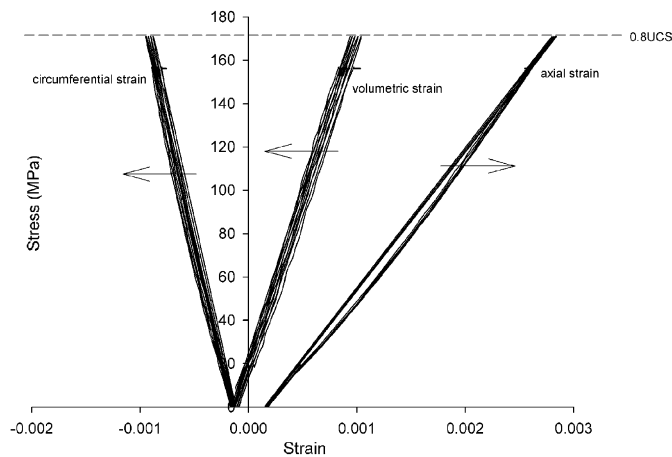


Fig. 6. Typical stress–strain curve for Westerly granite under constant-amplitude cyclic loading (sample WG-10). Arrows indicate how the axial, circumferential and volumetric strains change during the accumulation of microcrack damage during the experiment.

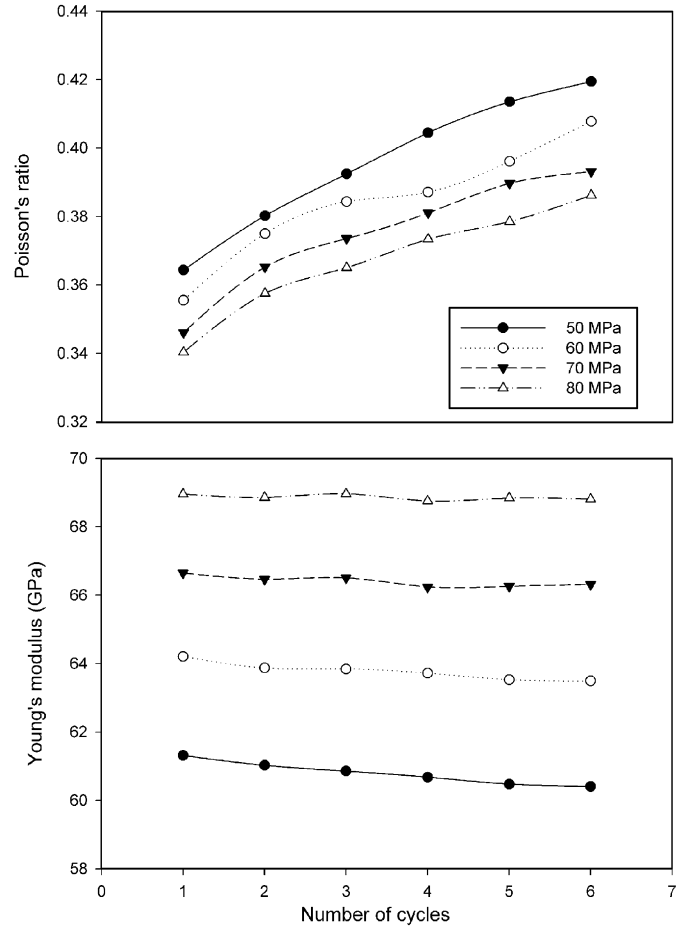


Fig. 7. Elastic property development in Westerly granite with increasing number of cycles during a constant-amplitude cyclic experiment (sample WG-10).

that time-dependent effects are almost negligible to the development of static elastic properties over the timeframe and conditions under which the experiments were conducted. The axial strain values showed no variation, within error, over any of the load-hold periods and immediately preceding stress cycles there are obvious ‘jumps’ in axial strain. Therefore, the accumulated damage seen in Fig. 7 is likely to be largely the result of large-amplitude stress cycling. Sample WG-13 validated this hypothesis and showed that, without cycles, little variation in axial strain damage is observed over the implemented experimental timeframe and conditions. Mechanisms operating during the constant-amplitude experiments have an overall modest effect on the evolution of static elastic properties.

#### 4. Discussion

Experimental rock mechanics has defined three types of microcracking. (1) Stress-induced microcracking, where the stress locally exceeds a fracture criterion, causing the crack to propagate spontaneously. Crack propagation is either stable or unstable. (2) Cyclic fatigue microcracking, where cracking is produced by the working of cracks as they open

and close in response to cyclic loading. (3) Time-dependent microcracking, where stable crack growth occurs below the critical stress intensity factor ( $K_{Ic}$ ), and are due to several competing factors of which the most familiar is stress corrosion [23,24,26,64,65]. Rock fracture is sensitive to which type is prevalent and mechanisms act simultaneously and independently [66].

In our experiments it is difficult or impossible to distinguish between the three mechanisms of microcrack growth. However we can say that the contribution of time-dependent crack growth is almost negligible to the development of static elastic properties under the implemented experimental timeframe and conditions for Westerly granite. Stress corrosion leading to time-dependent creep is undoubtedly taking place, but at an insufficient rate and magnitude required to observe its effect on the static elastic properties. A possible explanation for this is that the use of strain gauges restricted our experiments to dry conditions only. Most experiments designed to promote and observe time-dependent brittle creep are conducted under water-saturated conditions. Also, Westerly granite is a fine-grained, crystalline rock with a low porosity and low initial crack density and thus inhibits the transport of reactive species to the crack tips. Despite using significantly increased hold times than used in experiments by Lau and Chandler [17], we conclude the hold times need to be longer to observe the effect of time-dependent microcracking under our experimental conditions.

We attribute the gradual degradation in sample stiffness with respect to number of increasing amplitude stress cycles

(Fig. 5b) to the growth and coalescence of pre-existing and newly initiated microcracks. This is corroborated by thin section analysis. After eight large-amplitude stress cycles the rock is seen to contain many more microcracks (Fig. 8b) when compared with an undeformed sample (Fig. 8a). The microcracks are predominantly large intragranular and transgranular microcracks and display a preferred axial orientation (Figs. 8c and d). Fig. 5b shows that the Poisson's ratio increases exponentially just before failure (the last few cycles). As microcrack damage accumulates within the sample, predominantly in an axial crack orientation, then the circumferential strain increase becomes more prevalent relative to the decreasing axial strain. Hence the Poisson's ratio increases as crack damage is accumulated in the sample. The most plausible explanation for the stress-dependency of the Young's modulus shown in Fig. 5a is that the stress acts to close oblique and transversely orientated microcracks with increasing axial load; this can be related to the mean stress increase during loading. However, it must be noted that Westerly granite has a low initial crack density and Fig. 8 only shows a modest number of oblique and transversely orientated microcracks.

Our experimental data were collected under uniaxial conditions and the elastic property evolution may differ under a confining pressure [67]. Indeed, experiments have shown that the onset of dilatancy in Westerly granite is reduced by cyclic loading under uniaxial conditions but remain largely unaffected under triaxial conditions [68]. This implies that, under triaxial conditions, the effect of

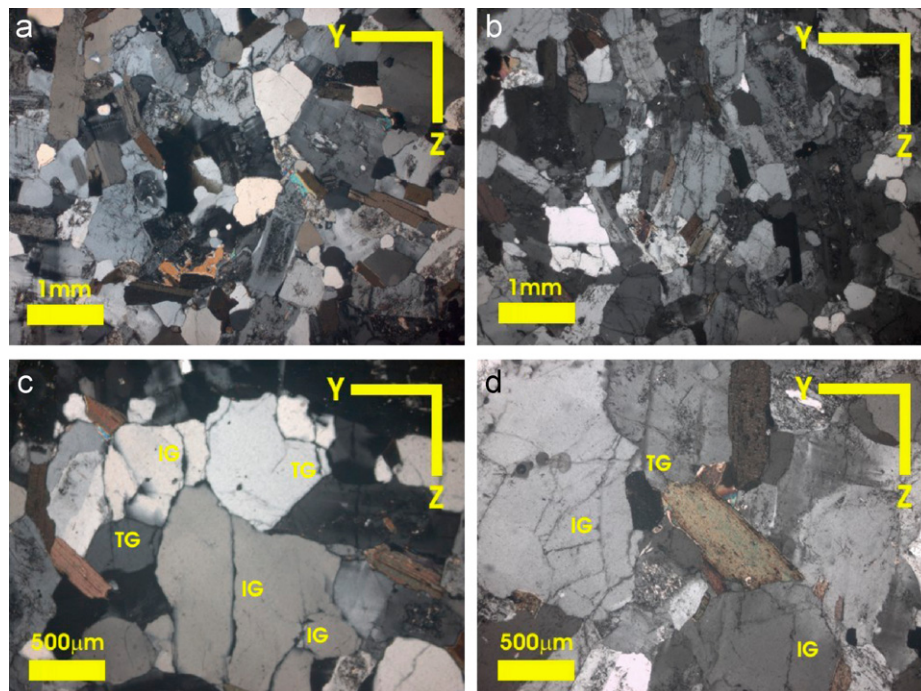


Fig. 8. Photomicrographs of Westerly granite under cross-polarized light. (a) undeformed sample. (b) Sample WG-08 after eight large-amplitude stress cycles. (c) and (d) Magnified photomicrographs of sample WG-08 after eight large-amplitude stress cycles showing large intragranular and transgranular microcracks in grains of quartz. Thin sections were taken along the YZ plane (see Fig. 1 for sample orientation) and axial compression was in the Z-direction. IG—intragranular microcrack; TG—transgranular microcrack.



stress-cycling on elastic moduli would be reduced. There are very few experimental studies of the evolution of rock elasticity prior to failure under a confining pressure. However, some dynamic elastic properties calculated from seismic velocity measurements from tests taken to failure at confining pressure are available for comparison. Ayling et al. [20] monitored dynamic elastic property changes in Darley Dale sandstone deformed at 50 MPa confining pressure. Although a different rock type was used in this experiment the changes seen in our results are of the same magnitude.

In nature, in addition to the microscopic damage imparted to our experimental samples, meso- and macro-scale fractures present within the Earth's crust will further reduce the overall bulk elastic properties of a rock mass. The effect of these larger scale features on elastic moduli is impossible to determine at the laboratory scale. Hence our data may be viewed as a 'minimum' effect that might be observed as it does not account for this additional effect. Finally, our data relate only to low porosity crystalline rocks whereas in nature deformation affects a wide variety of rock types that may possess different porosities, elastic properties or a mechanical anisotropy. However, such data are invaluable for an insight into the magnitude of static elastic property evolution with deformation and trends between the variables; and crystalline rock is commonly fractured at depth.

## 5. Conclusions

We have presented a new methodology to help identify perfect elasticity in stress–strain experiments. Previously, reliable data has only been obtained from the linear elastic portion of the stress strain curve. Our methodology enhances the range of stress values from which the elastic properties can be calculated and allows a detailed study into the evolution of the static elastic properties of rock as it approaches failure.

Our data show that significant changes in the static elastic properties occur as crystalline rock accumulates microcrack damage. If the elastic properties are monitored for a constant stress but as a function of number of increasing amplitude stress cycles, the changes seen equate to a decrease in Young's modulus of ~11% and a substantial increase in Poisson's ratio (~43%) before the rock reaches failure (Fig. 5b).

Our data indicate a strong stress-dependency of static Young's modulus (~46% increase) within a single loading cycle (Fig. 5a). At low loads there is an increase in Young's modulus in the region OA due to crack closure. The elastic properties stay constant in the linear elastic region AB. Above B there is an increase in Young's modulus until the behaviour becomes inelastic by exceeding the previous highest applied stress.

We were not able to distinguish between different modes of crack initiation and growth, such as stress induced microcracking or cyclic fatigue microcracking. However,

we are able to say that under the dry test conditions and timescales used in experiments, time-dependent microcracking plays a very minor role in the development of the elastic properties.

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