Scaling of Crack Surfaces and Implications for Fracture Mechanics

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The scaling laws describing the roughness development of crack surfaces are incorporated into the Griffith criterion. We show that, in the case of a Family-Vicsek scaling, the energy balance leads to a purely elastic brittle behavior. On the contrary, it appears that an anomalous scaling reflects an R-curve behavior associated with a size effect of the critical resistance to crack growth in agreement with the fracture process of heterogeneous brittle materials exhibiting a microcracking damage.

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Fractography has always been a useful tool for the understanding of fracture mechanisms in heterogeneous materials. It is well established that the fracture properties of brittle heterogeneous materials such as rock, concrete, tough ceramics, wood, and various composites strongly depend on the microstructure and on the damage process of the material. Both these parameters are also at the source of the roughness of crack surfaces. Indeed, microstructural heterogeneities acting on the crack front as obstacles, and the existence of a large fracture process zone with cracking damage where elastic interactions occur between cracks, may have a strong influence on the local deviations of the main crack [1–3]. Thus, the roughness of fracture surfaces can be considered as an inheritance of the heterogeneous character and of the damage process of the material. Hence, it is naturally tempting to correlate the fracture surface morphology to macroscopic mechanical properties such as fracture energy or toughness.

The studies of quantitative fractography are nowadays a very active field of research. Many experiments [4–11] on materials as different as ductile aluminum alloys to brittle materials like rock or wood have shown that the topography of fracture surfaces is self-affine [12]. However, in most cases and in spite of strong differences in the materials, a roughness index \( z_{\text{loc}} \approx 0.8–0.9 \) (called local roughness exponent in what follows) has been reported in all cases. This robustness of the results seems to support the idea suggested by Bouchaud et al. [6] that the local roughness exponent might have a universal value, i.e., independent of the fracture mode and of the material.

Nevertheless, if the local roughness exponent is universal, the range of length scales within which this scaling domain is observed strongly depends on the material microstructure. Thus, recent studies focused on the complete description (3D) of the crack morphology of granite [9] and wood [10] have shown that the scaling laws governing the crack developments in longitudinal and transverse directions are different and material dependent. Let us consider the development of a fracture surface from a border where a straight notch of length \( L \) imposes a zero roughness. The mean plane of the crack surface is defined as \((x, y)\) where the \( x \) axis is perpendicular to the direction of crack propagation and the \( y \) axis is parallel to this propagation direction. It has been found that the height fluctuations \( \Delta h \) of fracture surfaces of two heterogeneous brittle materials (granite [9] and wood [10]) estimated over a window of size \( l \) along the \( x \) axis and at a distance \( y \) from the initial notch exhibited intrinsically anomalous scaling properties obtained in some models of nonequilibrium kinetic roughening [13–16]:

\[
\Delta h(l, y) = A \left[ l^{z_{\text{loc}}(\xi(y))} z_{\text{loc}} - \xi_{\text{loc}} \right] \frac{\xi(y)}{l} \left[ l \ll \xi(y) \right], \quad \text{if } l \gg \xi(y),
\]

where \( \xi(y) = By^{1/z} \) depends on the distance to the initial notch \( y \) and corresponds to the crossover length along the \( x \) axis below which the surface is self-affine with the local roughness exponent \( z_{\text{loc}} \). Along the \( y \) axis, the roughness develops according to two different regimes: for large length scales, i.e., \( l \gg \xi(y) \), the roughness grows as \( \Delta h(l, y) \sim y^{1/\xi} \) where \( \zeta \) is called the global roughness exponent, while for small length scales, i.e., \( l \ll \xi(y) \), the roughness growth is characterized by the exponent \( \zeta - z_{\text{loc}}) \). The global roughness exponent \( \zeta \), the exponent \( z \) (called dynamic exponent), and the prefactors \( A \) and \( B \) are material dependent. Thus, despite the universal distribution of the height fluctuations in the transverse direction of crack propagation, the roughening in longitudinal direction is material dependent and has an influence on the roughness magnitude in the transverse direction. All lengths are dimensionless. Real lengths are divided by a unity length \( l^* \).

One main consequence is that, when the global saturation occurs, i.e., far from the notch \([y \gg y_{\text{sat}}] \), where \( y_{\text{sat}} = (L/B)^{1/z} \), the magnitude of the roughness is not just a function of the window size but also of the system size \( L \) : \( \Delta h(l, y \gg y_{\text{sat}}) \approx A l^{z_{\text{loc}}(\xi(y))} \). This unconventional dependence of the stationary local fluctuations on the
system size is distinctly different from what happens in
the Family-Vicsek scaling [17] which is defined as
\[
\Delta h(l, y) \approx A \begin{cases} \frac{1}{\xi_{\text{loc}}} & \text{if } l \ll \xi(y), \\
\frac{1}{\xi(y) \xi_{\text{loc}}} & \text{if } l \gg \xi(y). \end{cases}
\] (2)
The Family-Vicsek scaling can be seen as a particular case
of anomalous scaling where \( \xi = \xi_{\text{loc}} \).
In this Letter, we discuss the link between the rough-
ening of crack surfaces and the fracture process. On the
basis of the Griffith criterion, we show that a fracture
surface exhibiting an anomalous scaling reflects funda-
mental mechanical behavior of quasibrittle materials:
R-curve behavior (i.e., an evolution of the resistance to
 crack growth as a function of the crack length increment
[3]) and size effect of the elastic energy release rate.
Figure 1 shows an illustration of an R-curve behavior
observed for a wood crack propagation (mode I) using
a tapered double cantilever beam (TDCB) specimen
[10,18]. The energy release rate \( G \) (proportional to the
square of the toughness) evolves very significantly at the
onset of crack propagation and becomes independent of
the crack length after a characteristic propagation distance.
In Fig. 2, an example of size effect on this critical energy
release rate at saturation is presented [18].
Several connections between the fractal dimension of
the crack surfaces and the fracture energy or toughness
have been proposed [4,5,19–24]. Nevertheless, most
of the proposed analytical models [19–24] show weak
connections with the fracture behavior of brittle heteroge-
nous materials and especially with the phenomenological
R-curve behavior and size effect. However, these models
have been built on the basis of a 2D self-affine crack sur-
face disregarding the anisotropy of the scaling laws govern-
ing the crack developments in longitudinal and transverse
directions. In this case, either an R-curve is obtained with-
out any size effect or a size effect is obtained without an
R-curve. Let us consider a semi-infinite linear elastic
specimen of thickness \( L \) containing an initial crack at po-
sition \( \Delta a \) submitted to a uniaxial stable and slow tension
(mode I). In the classical Griffith approach, the fracture
criterion is estimated by balancing the elastic energy
released at the macroscale during an infinitesimal crack
propagation and the energy required to create the cor-
responding free surfaces at the microscale. According
to linear elastic fracture mechanics (LEFM), the rate of
energy dissipation in the structure as a whole must be
deﬁned with respect to the projected crack surface \( A_p \)
(at macroscale, crack roughness can be ignored). On the
contrary, the estimate of the energy required for crack
propagation at the microscale needs to take into account
the real crack surface \( A_r \). Introducing the elastic energy
release rate \( G \) at the macroscale, the fracture criterion
becomes for an infinitesimal crack advance
\[
G \delta A_p = 2 \gamma \delta A_r,
\] (3)
where \( \delta A_r \) is the real area increment and \( \delta A_p \) its projec-
tion on the fracture mean plane. The term \( \gamma \) is the so-called
specific surface energy that characterizes the resistance of
material to cracking. The real area can be obtained from
the surface description during the crack growth \( \delta a \) from
position \( \Delta a \) of the crack front: \( \delta A_r = \int_{\Delta a}^{\Delta a + \delta a} \psi(y) dy \),
where \( \psi(y) \) is the length of a profile at position \( y \) of the
crack roughness. For a specimen of width \( L \), a first or-
der approximation leads to the following expansion of the

![Figure 1](image1.png)

**FIG. 1.** R-curve behavior observed for a TDCB fracture ex-
periment in wood. For this Norway spruce specimen of charac-
teristic size \( L = 30 \) mm, data are fitted from Eqs. (5) and (6)
with the scaling exponent \( (\xi - \xi_{\text{loc}})/z \approx 0.32 \) which is consist-
tent with the expected result for this specimen: \( (\xi - \xi_{\text{loc}})/z \approx
0.36 \pm 0.09 \).

![Figure 2](image2.png)

**FIG. 2.** Size effect on the critical energy release rate \( G_{RC} \)
(Norway spruce specimens, \( 11.3 \leq L \leq 60 \) mm). The behavior
derived from the anomalous scaling (solid line) \( G_{RC} \sim L^{1-z_{\text{loc}}}
[Eq. (6)] \) is characterized by the exponent 0.69 which is in
good agreement with the expected result for spruce: \( \xi - \xi_{\text{loc}} =
0.73 \pm 0.17 \).
fracture criterion: $G = 2\gamma \psi(y)/L$. According to (1), the roughness $\Delta h(l, y)$ can be seen as a description of the fracture profile at position $y$. The length $\psi$ can be estimated by covering the profile path with segments of length $\delta$, the horizontal projection on the $x$ axis is $l_o$, and the vertical one on the $z$ axis is on average $[\text{see Eq. (1)}] A\xi(y)^{\xi-\xi_{\text{loc}}}l_{\text{loc}}$. The length $l_o$ is chosen as the lower cut-off of the fractal range (i.e., the characteristic size of the smaller microstructural element relevant for the fracture process). Moreover, the crack profile is a univalued function (observed fracture surfaces do not show overhangs) and hence the number of segments $\delta$ needed to cover the curve is given by the ratio $(L/l_o)$. As a consequence the real length $\psi = (L/l_o)\delta$ of the crack profile appears dependent on the distance $y$ to the initial notch and can be expressed as

$$\psi(y) = L \begin{cases} 1 + \left( \frac{A(B_{y}^{1/2}l_{\text{loc}}^{\xi-\xi_{\text{loc}}})}{l_{o}^{\xi-\xi_{\text{loc}}}} \right)^{2} \frac{\partial \psi}{\partial y} & \text{if } y \ll y_{\text{sat}}, \\ 1 + \left( \frac{AL_{y}^{\xi-\xi_{\text{loc}}}}{l_{o}^{\xi-\xi_{\text{loc}}}} \right)^{2} \frac{\partial \psi}{\partial y} & \text{if } y \gg y_{\text{sat}}. \end{cases}$$

In Eq. (4), the terms between brackets correspond to the tangent of the typical angle between the segment $\delta$ and the $x$ axis. Thus, as expected intuitively, the length of a self-affine curve is larger than its projected length although it is proportional to it. On the other hand, according to (4) when the global saturation state of the roughness is reached (i.e., for $y \gg y_{\text{sat}}$), the length of the crack profile saturates and becomes a nonlinear function of the specimen width $L$. Note that the length of the self-affine crack profile [Eq. (4)] is a finite quantity in contrast with the expressions proposed by Carpinteri [21], Borodich [23], and Bažant [24]. In these models, the actual length of a fractal curve is considered as infinite, and this leads to the use of an unconventional definition of fracture energy ($\text{fractal fracture energy}$). As we will show in the following, the introduction of a lower cutoff for the surface description allows one to obtain a fracture energy in agreement with the classical dimensions of LEMF.

In the zone where the roughness grows (i.e., for $\Delta a \ll \Delta a_{\text{sat}}$ with $\Delta a_{\text{sat}} = y_{\text{sat}}$), the fracture equilibrium (3) leads to an energy release rate function of $\Delta a$:

$$G_{R}(\Delta a \ll \Delta a_{\text{sat}}) = 2\gamma \sqrt{1 + \left( \frac{A}{l_{o}^{1-\xi_{\text{loc}}}} \right)^{2} \Delta a^{2(\xi-\xi_{\text{loc}})z}}.$$

In Eq. (5), the subscript $R$ emphasizes that the behavior of the resistance to fracture growth is similar to an $R$-curve [3]. When the crack increment is large, i.e., for $\Delta a \gg \Delta a_{\text{sat}}$ which corresponds to the saturation state of the roughness, the resistance to fracture growth becomes

$$G_{R}(\Delta a \gg \Delta a_{\text{sat}}) = 2\gamma \left[ 1 + \left( \frac{A}{l_{o}^{1-\xi_{\text{loc}}}} \right)^{2} \Delta a^{2(\xi-\xi_{\text{loc}})z} \right].$$

In the zone where the roughness saturates, the length of the crack profile remains constant which induces a resistance to crack growth independent of the crack length increment $\Delta a$ (6). The subscript $C$ in $G_{RC}$ emphasizes that the resistance to crack growth has reached an asymptotic or critical value. This post-$R$-curve regime at constant resistance to crack growth obtained here in an analytical form [Eq. (6)] is in agreement with the one generally assumed in the phenomenological approaches (for instance, [25]). In Fig. 1, Eqs. (5) and (6) are used to fit the data. Equation (5) provides a good description of the growth of the energy release rate. This fit needs to use an overestimated ratio $A/l_{o}^{1-\xi_{\text{loc}}}$. Note that a previous approach obtained from the energy released by a fractal pattern of microcracks ahead of the main crack of a quasibrittle material [23] has shown equally an $R$-curve behavior. On the other hand, it has been shown that the roughness of a crack induces a decrease of the stress intensity factor [26], and this shielding process is also known to be a source of the phenomenological $R$-curves. These latter points emphasize that the connection between the anomalous roughening of fracture surfaces and fracture mechanics seems to reflect the particular fracture behavior of brittle materials showing toughening mechanisms.

The main consequence of the anomalous scaling is the dependence of the critical resistance $G_{RC}$ to crack growth on the specimen size $L$ [see Eq. (6)]. Size effect phenomena are one of the main characteristics of the fracture

$$G_{RC}(\Delta a \gg \Delta a_{\text{sat}}) = 2\gamma \sqrt{1 + \left( \frac{A}{l_{o}^{1-\xi_{\text{loc}}}} \right)^{2} L^{2(\xi-\xi_{\text{loc}})}}. \tag{6}$$

In Fig. 3, theoretical size effect on the critical resistance to crack growth [Eq. (6)] obtained from an anomalous scaling and for the arbitrary scaling exponents $\xi = 1.3, \xi_{\text{loc}} = 0.8$, $A = 0.1$, and $l_{o} = 1$. 

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FIG. 3. Theoretical size effect on the critical resistance to crack growth [Eq. (6)] obtained from an anomalous scaling and for the arbitrary scaling exponents $\xi = 1.3, \xi_{\text{loc}} = 0.8$, $A = 0.1$, and $l_{o} = 1$. 

behavior of heterogeneous brittle materials such as concrete, rocks, or wood [25]. Figure 3 shows the theoretical evolution of critical resistance to crack growth as a function of the system size (or characteristic size) $L$ obtained for arbitrary values. It appears that the size effect shows two asymptotic behaviors, respectively, $G_{RC} = 2\gamma$ and $G_{RC} \sim L^{\zeta - \zeta_{oc}}$. From Eq. (6), one can define the crossover length $L_c = (l_0^{1 - \zeta_{oc}}/A)^{1/(\zeta - \zeta_{oc})}$. For small system sizes $L \ll L_c$, which correspond theoretically to “shallow” surfaces, i.e., surfaces that have a mean local angle of the crack profile smaller than $45^\circ$, there is no size effect: $G_{RC} \approx 2\gamma$. The roughness of crack surfaces being negligible, real crack surfaces are weakly different from the projected one, and so classical results of purely elastic fracture mechanics are recovered. For large system sizes $L \gg L_c$, the critical resistance evolves as a power law: $G_{RC} \sim L^{\zeta - \zeta_{oc}}$. These fracture surfaces are theoretically “spiky” (i.e., the average local angle of the crack profile is greater than $45^\circ$), and in this case, the critical energy release rates are greater than the specific surface energy $2\gamma$. Note that the latter power law behavior is in good agreement with the observed size effect in wood as shown in Fig. 2. In fact, the typical local angle of wood crack surfaces is smaller than $45^\circ$ but the crossover length $L_c$, calculated with the ratio $A/l_0^{1 - \zeta_{oc}}$ obtained from the $R$-curve fit (Fig. 1), is smaller than the sizes $L$ of tested specimens.

If the development of the roughness crack is driven by a Family-Vicsek scaling instead of an anomalous scaling, the crack profile length $\psi$ is independent of the crack increment $\Delta a$: $\psi(y) = \sqrt{1 + (A/l_0^{1 - \zeta_{oc}})^2} L$. Hence, the resistance to fracture growth reduces to

$$G_c(\Delta a) \approx 2\gamma \sqrt{1 + \left(\frac{A}{L_0^{1 - \zeta_{oc}}}\right)^2}.$$  \hfill (7)

Thus, for a Family-Vicsek scaling, the resistance to fracture is independent of the crack position $\Delta a$ and of the specimen size $L$. This is consistent with the behavior of purely elastic brittle materials. Likewise, if the global roughness exponent tends to the local one ($\zeta \rightarrow \zeta_{loc}$), the $R$-curve behavior (5) and the size effect (6) vanish progressively and the fracture behavior is close to purely elastic brittle materials [Eq. (7)].

In conclusion, on the basis of complete descriptions (3D) of fracture surface morphologies given by Family-Vicsek and anomalous scaling laws, we have shown that the connections between the roughening of fracture surfaces and fracture mechanics are important. A Family-Vicsek scaling of the fracture roughness reflects a purely elastic brittle fracture behavior, while the fracture behavior obtained on the basis of an anomalous scaling accounts for an $R$-curve behavior and size effect of the critical resistance to crack growth. Anomalous scaling reflects the experimental behavior of brittle heterogeneous materials showing toughening mechanisms. Ongoing experimental studies attempt to link systematically mechanical behaviors and fracture roughening for different materials.

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