Interfacial Crack Pinning: Effect of Nonlocal Interactions

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We propose a perturbative approach to describe the evolution of an interfacial crack between two elastic solids with quenched disorder. The driving force is the stress intensity factor along the crack front. The latter is expressed as a function of the entire crack geometry through a linear convolution with a long-ranged kernel using a first order approximation developed by Gao and Rice [J. Appl. Mech. 56, 828 (1989)]. The resulting problem is studied numerically and is shown to give rise to self-affine geometries with a roughness exponent $\xi = 0.35$ and a dynamic exponent $z = 1.5$, very different from the corresponding exponents obtained with a local form of the driving force.

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Following the pioneer work of Mandelbrot, Passoja, and Paullay [1], numerous studies have reported on the geometrical properties of crack surfaces, and shown, in particular, the self-affine properties of three-dimensional (see Ref. [2] for a recent review) and two-dimensional [3] cracks experimentally and numerically in discrete models of fracture [4,5]. Moreover, the observed roughness exponent appears to be universal [6,7]. The development of this roughness from a straight notch [8] has shown that the scaling tools developed for stochastic growth models [9] can be applied successfully. Moreover, recent studies [10] have focused on a theoretical analysis of the effect of heterogeneities in the crack plane on dynamic fracture propagation. However, no proper understanding of these properties has emerged yet.

Simultaneously, a lot of progress has been achieved to describe interface pinning by random impurities in the field of nonequilibrium statistical mechanics. A recent review [11] covers most recent developments. Therefore, it is natural to search for a connection between these two problems.

A first hint in this direction has been proposed recently by Bouchaud et al. [12] using a phenomenological approach. They proposed to see the crack surface as the trace left in space by the crack front during its propagation. The latter was proposed to be described by a nonlinear Langevin equation, keeping only the terms which are allowed by the symmetry of the system. Such an approach leads to a set of equations already considered by Ertas and Kardar [13] in a very different framework, namely, the de-pinning of vortex lines in superconductors. Unfortunately, the scaling exponents obtained in such a model are clearly different from those observed in fracture surfaces.

Two weak points can be identified in this interesting model. The first one is the nature of the noise, which in most cases should be described as a quenched noise (i.e., dependent on the space coordinate, but independent of time). In a simple Langevin equation, introduced and solved with an annealed noise by Edwards and Wilkinson [14], it has been shown recently [15,16] that if the noise is quenched, the scaling exponents of the model are significantly altered (the roughness exponent changes from $\xi = 1/2$ for an annealed noise to $\xi = 1.2$ for a quenched noise). The second criticism is about the assumption that the Langevin equation should be local (i.e., the rate of propagation of the crack at one point is only dependent on the crack geometry—position, slope, curvature,—at the same point). This is, in general, not justified within the context of fracture mechanics as we will see below, and again it can lead to drastic changes in the scaling exponents.

In this Letter, we propose to study a simpler problem, but using a more rigorous foundation. We consider an interfacial crack which propagates as shown schematically in Fig. 1 along a perfect plane $z = 0$, between two elastic solids having the same elastic properties but a higher toughness than the interface. No roughness is left behind the crack (the crack surface is $z = 0$). However, the crack front will acquire some turtuosity in the plane as it propagates if the interface is heterogeneous.

In order to characterize locally the heterogeneity of the interface, we introduce the local toughness $K_c(x, y)$...
as a quenched disorder. $K_c$ is assumed to be equal to a constant $K^0_c$ plus an uncorrelated fluctuation $\delta K_c$ of small amplitude with respect to $K^0_c$.

In the case of a straight crack front, the stress field in mode I is the well-known solution of Irwin [17]. It is a singular field which diverges as $K/(2\pi \sqrt{r})$ at a distance $r$ from the crack tip. If the crack front $y(x)$ is slightly distorted from a strictly linear geometry, Gao and Rice [18] have solved to first order in perturbation the stress intensity factor along the crack front. They have shown that $K(x)$ can be written as

$$\frac{K(x)}{K_0} = 1 + \frac{1}{2\pi} \text{p.v.} \int_{-\infty}^{\infty} \frac{y(x') - y(x)}{(x'-x)^2} \, dx',$$  \hspace{1cm} (1)

where $K_0$ is the stress intensity factor which would result if the crack was straight under the same loading conditions at infinity. Rice [19] has pointed out that this integral may be regularized and is defined in the principal value sense (p.v.) for a general perturbation by exploiting the known behavior of a crack under self-similar expansion, rotation, and translation. The necessary steps have been discussed by Bower and Ortiz [20], where a detailed discussion of the discretization procedure of Eq. (1) can be found. In practice, as long as the interface remains differentiable and its derivative Hölder continuous the problem is well posed.

It is worth noting an interesting parallel which can be drawn between the crack propagation problem and the quasistatic spreading of a partially wetting liquid on an heterogeneous plane. The displacement of a contact line is controlled by a local capillary force which is a nonlocal function of the entire contact line. Moreover, the functional dependence of the local capillary force on the contact line geometry has precisely the same form [21,22] as Eq. (1). This is in contrast with problems in a confined two-dimensional geometry (Hele-Shaw cell) for which the local capillary force may simply be considered as proportional to the local curvature of the contact line and therefore the kernel local. A qualitatively similar change of dimensionality would also be expected for a crack propagating between two thin plates.

From fracture mechanics, we know that the criterion for the propagation of the crack is $K \geq K_c$. Moreover, in order to simplify the model we assume that the propagation is quasistatic. To insure this condition, we introduce a simple rule for the propagation inspired from recent studies [15,16] on the role of quenched noise in some model Langevin equations. At each step of the computation, we look for the first site which will move if the loading is increased from 0. This site $x^*$ is such that

$$K(x^*) - K_c(x^*, y(x^*)) = \max_y [K(x) - K_c(x, y)].$$ \hspace{1cm} (2)

Once this site has been identified, the crack front is advanced locally by a random amount $dy$ uniformly sampled in the interval $[0,1]$, $y(x^*) \rightarrow y(x^*) + dy$. At the same time, the local toughness at this site is changed since it is assumed to be dependent on $x$ and $y$. Because the disorder is spatially uncorrelated, the new toughness is chosen randomly from a prescribed distribution chosen here to be uniform between 0 and 1.

Under this framework, the model has a mathematical structure comparable to the one studied in Ref. [16] for a different physical problem, beside a major difference in the expression of the driving force. In the latter reference, the equivalent to the stress intensity factor was a capillary force acting on a wettability heterogeneity in a 2D Hele-Shaw cell. Therefore, the driving force was simply proportional to the local curvature of the interface, and thus, focusing on the first order terms, the curvature was computed from the second derivative. We will see in the following that this leads to severe differences in the scaling properties of the model.

The geometry used in our numerical simulations is periodic along the mean direction of the crack front (i.e., the $x$ direction). This requires a resummation of the nonlocal kernel over all periods, but does not involve any difficulty. The discretization of the front is more subtle. It is imperative that the front has a continuous derivative. A naive discretization is, first, incompatible with the assumption of the perturbation computation, and, second, it leads to spurious unstable local modes where a single spike grows without limit. This latter effect is quite similar to the one observed in Ref. [23].

Let us now turn to a discussion of the results obtained in the numerical simulations. Two regimes appear successively as in most growth models [9]. First, the crack develops some roughness which increases with the crack mean...
position \( \langle y \rangle \). Then the roughness is limited by the system size, and it becomes statistically independent of \( \langle y \rangle \).

We first focus on this steady-state regime where the geometry can be characterized using the standard tools applied in growth models (see, e.g., Ref. [11]). A typical example of a front is shown in Fig. 2. The power spectrum of the front position \( P(k) \) averaged over 1000 samples, is shown in Fig. 3, and reveals a power-law behavior \( P(k) \propto k^{-1-2\xi} \), which means that the front is self-affine with a Hurst or roughness exponent \( \xi = 0.35 \pm 0.05 \). Such a result can also be confirmed through the analysis of the height deviation \( \sigma \) over varying size windows \( \delta \), where \( \sigma \approx \delta^\xi \). The latter method gives an estimate of \( \xi = 0.4 \pm 0.1 \), a value which is also confirmed by an analysis based on the return probability method. In the case where the interaction kernel is purely local—driving force proportional to the local curvature of the front—the roughness exponent is \( \xi = 1.2 \), almost three times as large as in the nonlocal case.

The time evolution in this steady-state regime is highly structured. The spatial distribution of active sites as a function of the mean crack advance displays a hierarchical cluster structure, which can be qualitatively compared to similar active sites in other growth models such as those reported in Ref. [16]. These correlations in the succession of active sites can be partly analyzed through the probability distribution \( p(d) \) of the distance \( d \) between two consecutive active sites, \( d = |x(t+1) - x(t)| \). Figure 4 shows that \( p(d) \) decreases as a power law \( P(d,1) \propto d^{-(b+1)} \), where \( b = 0.90 \pm 0.05 \). Again the \( b \) exponent in the model where the driving force is local is strongly different \( b = 1.87 \).

In order to model completely the scaling properties of the roughness, one needs two exponents. The first is the roughness exponent in the steady state, and the other—the dynamic exponent \( z \)—gives the time development of the front fluctuations. More precisely if \( \sigma(L) \) is the standard deviation of \( y(x) \) over the entire system size, one expects the following scaling:

\begin{equation}
\sigma(\bar{y}, L) = L^z \varphi \left( \frac{\bar{y}}{L^{1/z}} \right),
\end{equation}

where \( \bar{y} \) is the averaged position of the front.

At early stages \( \bar{y} \ll L^{1/z} \), the fluctuations of height are not dependent on the system size, and thus \( \varphi(x) \propto x^\xi \). On the other limit, for large times, the fluctuations \( \sigma \) are no longer dependent on the front position, so that \( \varphi(x) \propto x^\delta \).

This scaling has been verified by analyzing the early time evolution of the standard deviation \( \sigma(L) \) for different system sizes \( L \) from 32 to 1024, using, respectively, \( 10^4 \) to \( 10^5 \) independent realizations. The result is shown...
in Fig. 5 where the function $\varphi$ is plotted for all these cases. The good collapse obtained here allows the accurate determination of the scaling exponents, $\gamma = 1.5 \pm 0.1$ and $\zeta = 0.35 \pm 0.05$. The latter exponent is in perfect agreement with the previous analyses of the Hurst exponent in the stationary regime. This leads to $\varphi(x) \propto x^{0.35}$ in the early time regime, which is well observed in Fig. 5.

This study underlines the importance of an infinite medium surrounding the interface where all activity takes place. This medium has the only role of mediating the interactions between different regions of the front, giving rise to a nonlinear dependence of the driving force on the front geometry. We have shown that this nonlocality which can be written analytically to first order perturbation for a rough crack front does drastically alter the scaling properties of the front geometry, and hence it cannot be neglected. We have mostly considered an elastic crack problem, and mentioned the analogy with a wetting problem in three dimensions, but the problem at hand is much more general, and it can be extended to other cases as soon as the Green functions for the homogeneous problem are available.

The experimental analysis of the conformation of a meandering crack front has not yet been reported in the literature to our knowledge. The results presented in this Letter should stimulate experimental studies; one is actually in progress by the authors.

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